The Rise of Trade Volumes, the 'Origin-Margin', and Per Capita Income

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Abstract

Recent empirical literature documents the existence of an *origin-margin* in international trade: importers tend to increase the number of source countries per imported good. Along various dimensions, this trend for diversification strongly correlates with the increase in trade volumes. Existing explanations of the rise of world trade volumes cannot account for that parallel development. This paper jointly addresses the rise of trade volumes and the expansion of the origin-margin and suggests per capita income as a common determinant of both trends. It develops a model where varieties - defined as goods differentiated by origin - are non-essential in an otherwise standard love-for-variety utility. In presence of transportation costs consumers demand varieties from a strict sub-set of supplier countries. This sub-set expands as per capita income grows. The additional margin in the import-bundle induces milder decreasing returns from imported than from domestic varieties. Consequently, income growth shift expenditure shares towards imports and trade volumes rise. An additional section tests the paper's key predictions and presents a calibration exercise.

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1 Introduction

A good decade ago Krugman (1995) observed a surprising disagreement regarding the answer to the simple question "Why has trade grown?". As the most common explanations he listed, first, reductions in transportation costs, second, liberalized trade policies, and third, income convergence. In an attempt to settle the issue, Baier and Bergstrand (2001) empirically disentangled the impact of these three factors on the rise of world trade between the late 1950ies and the late 1980ies, showing that tariff cuts constitute by far the most important contribution, followed by a drop in transport cost and a negligible contribution of income convergence.

But with tariff cuts as the accepted prime force behind the rise in postwar trade shares the next puzzling observation emerged: In standard trade models the spectacular rise in trade volumes and the relatively modest tariff reductions imply an import elasticity exceeding all conventional estimates. Addressing this *elasticity-puzzle*, Yi (2003) shows that under realistic substitution elasticities between home and foreign goods, a modest fall in tariffs can cause strong responses in trade volumes if it generates international vertical specialization. Under vertical specialization, the value of a good enters the trade statistics many times as it is shipped back and forth at different production stages. With a calibrated model Yi (2003) explains up to half of the rise in world trade volumes. In an approach that stresses the dynamic effects of tariff cuts, Cuñat and Maffezzoli (2006) point out that trade liberalization change the incentives for factor accumulation in a Heckscher-Ohlin model. This leads to diverging factor endowments, deepens the comparative advantage, and thus raises trade volumes. The cumulated effects of gradual and anticipated trade liberalization are shown to generate sizable long-run responses that replicate the growth of US trade volumes. Finally, Ruhl (2003) suggests that the elasticity-puzzle originate in the different ways to measure elasticities. Firms that face fixed costs of export, he argues, react differently to permanent and to transitory price shocks. Unless transitory shocks, permanent tariff cuts increase the discounted flow of future profits from export markets considerably so that more firms start to export. Consequently, estimated elasticities are higher when identified through tariff reductions and lower when derived form high-frequency import data.

These approaches greatly help to understand the responses of trade volumes to tariffs and are rather successful in explaining the elasticity-puzzle. In order to differentiate between their fit of trade data, it seems therefore necessary to evaluate how they square with empirical trends other than sheer trade volumes. A recently documented trend seems particularly appropriate for this purpose as it closely parallels the growth of world trade volumes: the increase in the number of source countries per imported good or the growth of trade along the *origin-margin*. Broda and Weinstein (2004a) report a substantial expansion along the origin-margin for each of the 20 largest importers, thereby extending their analysis of the US from earlier work (Broda and Weinstein (2004)). This trend of international diversification appears interesting on its own but it is all the more important as it moves closely together with the rises in trade volumes. Based on two different datasets, Figure 1 illustrates this parallel increase of US import shares and the origin-margin. The impressive rise in US imports came along with a steady expansion of the average number of source countries per good. A closer look at these data supports the view that import volumes and the number of source countries co-move. For US data disaggregated by good categories (HTS and TSUSA) Figure 2 shows a strong correlation between changes in both variables from 1972 to 2000. Finally relying on bilateral trade data of 188 countries by 4-digit SITC good categories, Figure 3 repeats this plot adding a country dimension. Again, the graph exhibits a strong correlation between changes in import volumes and changes in the origin margin.

How well do the existing models square with this tight link between increases in trade volumes and in the origin margin? Unfortunately, they do not do particularly well. The one multi-country model (Ruhl (2003)) predicts rises in trade volumes exclusively along the intensive margin. Clearly, the two-country models cannot explain the origin-margin at all and also taking their predictions with a grain of salt, the difficulties do not disappear. The endogenous deepening of comparative advantage that drives trade volumes in Cuñat and Maffezzoli (2006), runs against the trend of international diversification documented with the expansion along the origin margin. The success of vertical specialization (Yi (2003)) in explaining the origin-margin generally depends on the classification of goods, in particular whether intermediate goods are kept in separate categories (in which case the model cannot account for the origin-margin) or are lumped together with the final product (in which case it can). As the very detailed HTS and TSUSA categories generally specify intermediate products separately, one would expect less variety growth on higher aggregation levels under the presumption that vertical specialization is the main force behind the growth in trade volumes.[2] However, Figure 3d shows that there is no such trend in the data and casts doubt on this explanation as the prime force of the rise in trade volumes.

In sum, the most important models that explain the growth in world trade volumes and the elasticity-puzzle do not perform well when it comes to the striking correlation between increases in trade volumes and the expansion of imported varieties. The present paper jointly addresses both of these trends. It develops a model that builds on the standard assumption that different varieties enter consumers' utility with finite substitution elasticity but it departs from the conventional framework

[2] E.g. the codes 85299039 and 85299039 stand, respectively, for "Parts of television receivers [...]" and "Combinations of parts of television receivers [...] for television apparatus other than television cameras", which represent different production stages in the spirit of Yi (2003). Both goods are lumped together on a 5-digit aggregation level. They are further combined with "Transmission apparatus for television" (code 85251020) on a 3-digit aggregation level. In a process of vertical specialization, each production step is performed by a single country such that there is complete specialization at highest aggregation levels and a trend to diversify at higher ones.

by assuming that varieties are non-essential to consumers. This latter assumption is a crucial one and drives the main results. It immediately implies that in presence of non-negligible trade costs countries import only from close neighbors and there is a marginal trade partner at a maximal shipping distance. This, in turn has two main consequences. First, income growth drives up the set of imported varieties since the marginal utility of those varieties that are already in the consumption basket drops, inducing consumers to purchase "new" varieties they did not consume before. Second, income growth increases trade volumes more than proportionally as a richer consumer spends a higher share of her income on the increasing number of imported goods. Technically speaking, the origin-margin constitutes an additional dimension to allocate an extra dollar of expenditure. This margin is present in the import bundle but absent in the bundle of domestic varieties. Thus, the utility derived from imports exhibits milder decreasing returns than the utility derived from domestic varieties and income growth translates into a rising marginal propensity of expenditure on imported goods, finally causing a rise in trade shares.

There is a number of secondary implications which are worth to mention and possibly merit to be pursued further. First, the models predicts that the tariff-equivalent trade cost increase in per capita income, very much unlike conventional models with homothetic utility function. At the same time, an exogenous drop in trade cost induces consumers to purchase varieties from more distant locations or equivalently, goods with higher trade costs from the same location. Both effects distort the measured trade cost from the actual one, which generally understates the drop in trade costs. Second, the present paper's model predicts that a country purchases only a subset of varieties traded worldwide. Haveman and Hummels (1999) show that this holds true for the US and a large set of other countries and conclude that "importers purchase a very small fraction of available varieties". The authors wonder "whether our models accurately characterize the way that firms differentiate and the way that consumers value that differentiation". This observation urges Anderson and van Wincoop (2003) to admit that an "important drawback of the existing theory is that all countries import all varieties from all countries" when referring to the class of multicountry models typically used to study the gravity equation. The present paper constitutes progress in this dimension. Third, if consumers marginal utility from varieties is actually bounded, the implied consumers surplus from an additional variety is less than that implied by CES utilities. In this case, estimates of consumer's surplus that assume utilities of constant substitution-elasticity as in Broda and Weinstein (2004) and (2004a) overstate the positive effect of expansions of new varieties and are possibly severely upward biased.

To document the quantitative performance of the model, a basic calibration exercise is performed. An central aspect of the calibration is to target the import elasticities since, most obviously, it is central to get the key variable right when addressing the elasticity-puzzle of international trade. In the present framework this is a particularly delicate issue since a bounded marginal utility implies unbounded ranges of the price elasticities at zero consumption levels. The calibration shows that at realistic elasticities the models is reasonably successful in replicating the time series from Figure 1. To further strengthen the paper's argument, a version of the models is used to derive predictions on bilateral trade flows and test them in the framework of the gravity equation. In an approximation around the symmetric equilibrium of the model, the usual elasticities of trade volumes to total GDP are slightly reduced. More importantly, the gravity equation is augmented by per capita income of the importer and the exporter country. A series of tests reject the null hypothesis of zero impact of per capita incomes on bilateral trade flows with the predicted positive signs and within the predicted range.

The current paper is not the first one to explore the link between trade and per capita income. Its closer predecessors are models of Markusen (1986) and Bergstrand (1990) who formalize and investigate on the so called "Linder hypothesis", which predicts rising intra-industry trade shares with per capita growth. These models assume that consumers cover a minimum level of an homogenous, domestically produced good before demanding imported "luxury" goods. The present model does not rely on this set of strict and questionable conditions. Instead, it builds on the assumption that all varieties enter the consumers' utility symmetrically and only transportation costs create asymmetries in demand via their effect on consumer prices.

The remainder of the paper is organized in the following way: section 2 develops a partial equilibrium model to illustrate the key mechanisms and offers a preview of results. Section 3 analyses the key mechanisms in a general equilibrium framework and derives the main results. Section 4 derives and tests an augmented gravity equation and presents some calibration results for the US trade data, and section 4 concludes.

2 A Partial Equilibrium Model

In this section, commodity prices are taken as given and the analysis concentrates on the consumer's choice. This approach clarifies the key mechanisms and predictions of the paper, which will then be studied in a general equilibrium context in a subsequent section.

Consider a consumer who disposes of a budget that she entirely spends on a consumption good. This consumption good is differentiated in a number of varieties, which are imperfect substitutes for the consumer. The consumer's preference structure will be represented by a utility function that exhibits the standard regularity conditions, i.e. differentiability, monotonicity, and concavity. In addition to these standard assumptions, the marginal utility derived from each of the varieties is supposed to be bounded. This latter feature constitutes a major departure from standard literature and is central to the results below. Preference structures with these characteristics have been used to study topics in trade literature and it will be convenient to follow an important precursor and assume with Young (1991) that the utility function takes the form

$$U = \int_o^\infty \ln (c_i + 1) \; di \tag{1}$$

where c_i is the consumption level of the variety i and $[0, \infty)$ represents the unbounded set of varieties. A noteworthy feature of this utility is that all varieties enter the consumer's preferences symmetrically so that consumption levels of all varieties are identical if and only if prices are. Thus, varieties are a priori identical to the consumer and it is possible to order them without loss of generality in such a way that prices are increasing in the index, i.e. that

$$i < j \quad \Rightarrow \quad p(i) \leq p(j) \qquad\qquad \forall \quad i, j \in [0, \infty) \tag{2}$$

holds.

The consumer's optimization problem is now to maximize (1) subject to the budget constraint

$$\int p_i c_i \, di \leq E \tag{3}$$

where E represents the consumer's wealth. Normalizing the first variety's price to unity, $p_o = 1$, the optimality conditions imply

$$\frac{c_o + 1}{c_i + 1} \leq p_i \tag{4}$$

where equality holds whenever $c_i > 0$. Assuming that there is no upper bound on the prices ($\lim_{i \to \infty} p_i = \infty$), the consumer with the bounded marginal utility from each variety does not consume all varieties but a strict subset of the cheapest varieties. In that case, there is an cut-off variety with index ρ that satisfies

$$c_i = 0 \qquad \Leftrightarrow \qquad i \geq \rho$$

Combining equations (3) and (4) and taking implicit derivatives leads to

$$\frac{d\rho}{dE} = \frac{1}{\rho p'_\rho} > 0 \tag{5}$$

which proves the following

Proposition 1 *Given that the price schedule p_i is differentiable at $i = \rho$, the set of varieties consumed by an individual increases with her per capita income. The increase is larger, the smaller is the derivative $dp_i/di|_{i=\rho}$ and the smaller is ρ, the number of varieties already consumed.*

The important part of the statement is the positive link between per capita income and the size of the consumption basket. The intuition for this result is straight forward. An individual who receives an increase in income can purchase more of each variety she previously consumed, which makes marginal utility derived from these varieties drop. At the same time, the marginal utility from the varieties not consumed stays constant such that the ratios of marginal utilities shifts in favor of the varieties not consumed. This means that the varieties in a small neighborhood of ρ, which were not consumed previously now look relatively more attractive and enter the consumption bundle. In order to make (4) hold, these marginal varieties enter the consumption bundle are consumed now.

The present paper will extensively study the expenditure shares of subsets of varieties, in particular imported ones. It is therefore convenient to write the expenditure on a subset of varieties $S \subset [0, \infty)$ as

$$E_S = \int_{S \cap [0,\rho)} p_i c_i \, di = \mu(S \cap [0, \rho))(c_o + 1) - \int_{S \cap [0,\rho)} p_i \, di$$

where μ is the standard measure on $\mathbb{R}$. One can now show the following

Proposition 2 *Total expenditure on each of subset S of varieties increases in total income E. Moreover, the expenditure share E_S/E on a subset of varieties S increases if and only if the average price of the set of varieties S exceeds the average price of the set $[0, \rho]$.*

Proof. Use binding condition (4) to see $dc_o/dE = 1/\rho$ such that $dE_S/dE = \mu(S \cap [0, \rho))/\rho$. Since $\int_{S \cap [0,\rho)} p_i \, di / \int_o^\rho p_i \, di > \mu(S \cap [0, \rho))/\rho$ this proves the statement. ■

Comparing varieties it is clear that a higher price of a variety means a smaller expenditure share with the marginal variety with index ρ having a share of zero. Starting from zero, this share increases fastest compared to others. Equivalently, a variety with a higher price increases its share faster than a variety with lower price. The proposition confirms that this intuition also holds true for the average varieties of subsets of varieties.

With the results phrased in Propositions 1 and 2, one can already venture a tentative interpretation of the parallel rise of trade volumes and of the number of source countries per good documented in Figures 1 to 3. Suppose a country is populated with identical individuals of the kind described above. The one good of the world economy comes in different varieties, part of which are produced abroad, the rest is produced at home. Foreign varieties have to be shipped across oceans and borders in order to be consumed at home so that, as such transport is costly, foreign varieties tend to be pricey compared with the otherwise identical domestic varieties. Finally, there may be closer and more distant trade partners which can export their varieties at lower and higher transport cost, respectively. These assumptions imply that the higher priced varieties tend to be the foreign ones and among the foreign varieties, the ones produced at distant locations tend to be more expensive.

Now, according to Proposition 1, a rise in per capita income expands the set of varieties consumed and consequently raises the number of imported varieties. (Indeed, the number of imported varieties increases by more than the number of domestic varieties). Further, and according to Proposition 2, the expenditure share on imported varieties increases in response to a rise in per capita income because the bundle of imported varieties is, on average, more expensive. Thus, the country's trade share and the number of varieties it imports move together in response to variations in the underlying determinant, per capita income of its inhabitants.

These plain implications of a very simple model that can be addressed in a first basic test. The International Trade Data provided by NBER-UN report bilateral trade disaggregated by 4-digit SITC sectors for an almost complete coverage of countries (see Feenstra (2005)). In order to test the first proposition, standard practices are applies and the goods sectors are differentiated into varieties by virtue of the country of origin in order to calculate average number of varieties per good for the initial and the end period. The change in LOG of this ratio is then regressed on the change in LOG of per capita GDP for the period 1972 to 2000. To control for the well-known predictions of the gravity equation (imports are proportional to total GDP), changes in population are added to the set of independent variables as well as initial levels of the dependent variable.

Table 1a reports the results for OLS regression including different sets of the independent variables. In all of the specifications the null hypothesis that a change in change in per capita GDP does not affect the change in the dependent variable is rejected on all conventional significance levels and the estimated coefficients are positive. Note that the estimates of population growth are negative and significant in the three specifications. This is an interesting result for which the partial equilibrium model above does not offer an explanation; but it will be discussed in detail in the general equilibrium analysis further below. To test the statement of Proposition 2, a parallel test is performed with the LOG change in import volumes as the dependent variable. The results are reported in Table 1b. Here again, the estimated coefficients of the change in LOG income per capita are positive and the confidence level is 1% throughout.

Finally, it seems natural to look at the impact of trade costs on the patterns of trade since trade cost plays a key role in the mechanism sketched here. Proposition 1 provides a testable implication of the shape of the price schedule and thus of the nature of transportation cost. Other things equal, a steeper rise in transportation cost should lead to a steeper rise along the price schedule ordered according to the rule (2). The US Import and Export Data provide detailed information on imports by good and source country and include imports duties and charges (see Feenstra et al (2002)). These data can be used to calculate tariff-equivalent transportation cost τ_k within each sector k, which is the used to estimate the slope of the transportation cost along ordered varieties by estimating the equation

$$\ln(\tau_{i,k}) = \alpha_k + \gamma_k r_i$$

where r_i is the rank of the ordered variety within a sector. To obtain a larger number of observations, HTS- and TSUSA-categories are lumped together to sectors defined by the respective 2-digit codes. Outliers are eliminated. The estimates of $\hat{\gamma}_k$ are then used to regress the change in the number of varieties per good on the slope of trade cost, i.e. to run a regression of the form

$$\Delta \ln(n_k) = \alpha + \beta \hat{\gamma}_k$$

where n_k is the average number of varieties per good in sector k and Δ indicates the change over the periods 1972-1988 and 1989-2000. The $\hat{\gamma}_k$ are estimated using data at the end of the respective period. Table 2 reports the results of the OLS regressions. Consistent with Proposition 1 higher slope of trade costs (δT.Cost) is estimated to affect negatively the increase in the number of varieties per good. Unless the success of this variable, the number of goods and varieties per good at the initial date of the periods, however, are positive, although not always significant.

While the simple model developed in the present section delivered useful insights and the consecutive tests gave some promising results, a number of key issues remain unsolved. Most obviously, a thorough analysis of trade patterns must include endogenously determined prices. In particular, the demand effects formulated in Propositions 1and 2 will involve general equilibrium aspects that impact a country's terms of trade and will not leave the results unaffected. Further, as Figure 1 shows, the rise in the number of varieties per good come along with a rise in the number of traded goods. Addressing the first while holding the second constant is not very satisfactory. Lastly, proper predictions on bilateral trade flows are desirable in order to compare them with the standard predictions of the so-called gravity equation. In order to address these issues, a general equilibrium analysis will be performed in the following section.

3 A General Equilibrium Model

There is a continuum of countries, located on a circle C. Country $i \in C$ is populated by identical individuals, which inelastically supply labor L_i to the domestic labor market. For the time being and until specified otherwise, there is only one good. This good is differentiated by the origin of production (the Armington assumption); following standard terminology, output from two distinct source countries will represent two different varieties of the good.

The good produced in country i is further differentiated by different types V_i. Deviating somewhat from the standard terminology, these types are equally called varieties throughout the paper. Thus, varieties are defined along two dimensions: within a country and by geographical origin.

Production. Country i produces competitively its varieties V_i with labor as the sole factor according to the linear production function

$$x_{ik} = a_i L_{ik} \qquad\qquad k \in V_i \quad i \in C \tag{6}$$

Note that productivity a_i is allowed to differ by country but not by variety. Finally, the varieties V_i are exogenously partitioned in tradables and nontradables. In particular, there is one single tradable and the set N_i of nontradables[3] with mass α. Thus, the total set of varieties available to consumers in country i can be identified with $N_i \cup C$.

Preferences. Consumers value all - tradables and nontradables - varieties symmetrically. The preferences are summarized by the utility function

$$U_i = \int_{N_i \cup C} \ln(c_{ik} + 1)\, dk \tag{7}$$

where c_{ik} is the quantity of variety k consumed by an individual in country i. As a most important deviation from a standard setup, this utility departs from the familiar Cobb-Douglas structure in the unit constant added to consumption level[4]. Consequently, marginal utility of each variety has a finite upper bound so that consumers refrain from consumption of varieties whose prices exceed a maximum.

Note also that utility (7) is separable in the parts derived from the imported and the nontradable varieties:

$$U_i = u_N + u_C \tag{8}$$

with the sub-utilities

$$u_X = \int_X \ln(c_{ik} + 1)\, dk \qquad\qquad X = N, C \tag{9}$$

Utility maximization can therefore be divided to a first stage involving sub-utilities (9) given expenditure on imports and domestic varieties, and a second stage where these expenditure levels are determined by maximizing (8). Before looking at consumers' optimal choices, however, the description of the model's structure will be completed.

Prices. Producer price p_j in country j and the corresponding consumer price q_{ij} in country i and are assumed to differ due to positive trade costs that drive a

[3] This is an artificial but innocent assumption. It guarantees that the trade share of the atomistic countries is strictly smaller that one. Technically speaking, the set of tradables and nontraded types must have equal cardinality.

[4] See e.g. Young (1991) for a motivation of the deviation from the standard.

wedge between both. In particular, the relation between q_{ij} and p_j is assumed to be of the following functional form

$$q_{ij} = \begin{cases} p_j T_{ji}(1+\delta r_{ij}^{\gamma}) & \text{if } i \neq j \\ p_i & \text{if } i = j \end{cases}$$

where r_{ij} is the distance between country i and country j. The positive parameters γ and δ imply that the standard "iceberg" transportation cost is increasing in distance. The gross border cost $T_{ji} \geq 1$ on goods exported from j to i can be thought of as a cumulative effect of tariffs, delays at borders, exchange rate risk and the like. Some of these costs enter prices additively, while others effectively work as a tax on consumer prices, multiplying prices. In the current setup, T is assumed multiply prices - a convenient but inessential assumption. Tariff revenues, as far as they are generated, are burned.

For tractability of the model assume symmetry across "almost all" countries. It will prove useful to focus the analysis on one single country, indexed with 0, that differs in its parameters from the set of identical rest of the world countries $i \in C \backslash \{0\}$. Consequently, prices and quantities that country 0 imports can be indexed conveniently by the distance r between a ROW-country and country 0

$$q_r = \begin{cases} p^* T(1+\delta r^{\gamma}) & \text{if } r > 0 \\ p & \text{if } r = 0 \end{cases} \tag{10}$$

where here and in the following, ROW-variables will be marked with an asterisk.

1^{st} Stage Optimization. The set of imported varieties and optimal bundle generally depends on all world prices. If trade costs are negligible ($T = 1$, $\delta = 0$) all imports to country 0 have the same price and every consumer in country 0 purchases the full bundle of varieties available on the world market. Yet, when trade is costly, consumer prices of varieties differ due to the trade costs. Let the representative individual in country 0 spend the total amount I_m (in domestic currency unites) on imported varieties. The optimal bundle of imports then maximizes u_C from (9) subject to the budget constraint

$$\int q_r c_r \, dr = I_m \tag{11}$$

under the price schedule (10). The optimality conditions give relative demand for the varieties imported from two countries at distances r and r'

$$\frac{c_r + 1}{c_{r'} + 1} = \frac{q_{r'}}{q_r} = \frac{1+\delta r'^{\gamma}}{1+\delta r^{\gamma}} \tag{12}$$

whenever c_r and $c_{\tilde{r}}$ are positive.

For country 0 import prices are strictly increasing with distance. Thus, there is a "marginal exporter" at distance $\overline{r}$, from which country 0 imports a negligible amount provided that l the circumference of the circle C is large, i.e. $\overline{r} < l/2$. This last condition is assumed to hold for the rest of the paper, implying that the maximal distance a variety is shipped is less than the maximal distance between two countries on the circle.

Equation (12) can now be rewritten in terms of $\overline{r}$ such that for all foreign varieties domestic consumption is

$$c_r = \max\left\{\delta\frac{\overline{r}^{\gamma} - r^{\gamma}}{1 + \delta r^{\gamma}}, 0\right\} \tag{13}$$

The value $\overline{r}$ is equal to half of the mass of different varieties that country 0 actually imports and will turn out to be a central variable in the following. $\overline{r}$ can be expressed in terms of the expenditure I_m, producer prices p^*, and the parameters by using equations (10), (11), and (13)

$$\overline{r} = \left(\frac{I_m(1+\gamma)}{Tp^* 2\gamma\delta}\right)^{1/(1+\gamma)} \tag{14}$$

Equation (14) shows that the $\overline{r}$ is an increasing function of I_m: at constant world prices, the mass of imported varieties increases with the expenditure on tradables varieties. Not only more of the same varieties, but also new varieties are imported when the expenditure on imports rises, i.e. the bundle of traded varieties has a non-trivial origin-margin.

With the help of equation (13), the sub-utility of the imported bundle (9) can also be expressed in terms of $\overline{r}$:

$$u_C = 2\overline{r}\ln(1 + \delta\overline{r}^{\gamma}) - 2\int_o^{\overline{r}} \ln(1 + \delta r^{\gamma})\, dr$$

such that $du_C/d\overline{r} = 2\gamma\delta\overline{r}^{\gamma}/(1 + \delta\overline{r}^{\gamma})$. This, together with (14) leads to the marginal sub-utility derived from expenditure on the optimal tradable bundle

$$\frac{du_C}{dI_m} = \frac{du_C}{d\overline{r}}\frac{d\overline{r}}{dI_m} = \frac{1}{Tp^*\left(1 + \delta\left(\frac{I_m(1+\gamma)}{Tp^* 2\gamma\delta}\right)^{\gamma/(1+\gamma)}\right)} \tag{15}$$

The important feature to note from equation (15) is that the expenditure on tradables I_m enters the denominator of the marginal sub-utility with the power of $\gamma/(1+\gamma)$, such that the decreasing returns to I_m is milder, the smaller is the parameter γ.

The optimal bundle of domestic varieties is quickly determined. When I is total per capita expenditure, the expenditure of the representative individual on domestic varieties is $I_d = I - I_m$. As all nontradables are identical, one has $\alpha p c_d = I - I_m$

where c_d is the the quantity consumed of the representative domestic variety. Thus, the individuals' utility (9) from nontradables is $u_N = \alpha \ln(1 + c_d)$ and

$$\frac{du_N}{d(I - I_m)} = \frac{\alpha}{\alpha p + I - I_m} \tag{16}$$

It is worth to note a qualitative difference between marginal utility from expenditure on tradable (15) and nontradable (16) varieties. The respective expenditures enter the denominator of du_D/dI_d linearly, but only sublinearly the denominator of du_C/dI_d. In other words, the marginal utility of imported bundles is decreasing in expenditure at milder rates than marginal utility of the domestic bundle. This feature is due to the origin-margin in the imported varieties and, as will be shown shortly, implies that the import share rises with per capita income.

2^{nd} Stage Optimization Since all income is allocated to consumption each period, total income of an individual in country 0 equals its its total expenditure is $I = pa$. Writing e as the share of income spent on imported varieties one has $I_m = pae$ and $I - I_m = pa(1 - e)$ and thus

$$c_d = a(1 - e)/\alpha \tag{17}$$

Note that e is also the trade share of country 0 since the value of total imports over value of total output in country 0 is $I_m L/(paL) = e$. It will be useful to rewrite (14) and express the trade share in terms of $\bar{r}$

$$e = \frac{2\gamma}{\gamma + 1} \delta T \pi \frac{\bar{r}^{\gamma+1}}{a} \tag{18}$$

where here and in the following $\pi = p^*/p$ stands for the inverse of country 0's terms of trade.

The 2^{nd} stage optimization simply requires $du_N/dI_d \geq du_C/dI_m$ where the inequality binds whenever trade volumes are positive. With equations (15), and (16) is is quick to check that e is positive whenever condition

$$T\pi - 1 < a/\alpha \tag{19}$$

holds. This condition will is assumed to be satisfied in the following. By imposing the optimality condition $du_N/d(I - I_m) = du_C/dI_m$ and using equations (14) - (18), $\bar{r}$ is implicitly determined as a function of world prices and the underlying parameters by

$$\alpha T \pi \left(1 + \delta \bar{r}^{\gamma}\right) + \bar{r}^{\gamma+1} T \pi \delta \frac{2\gamma}{\gamma + 1} - \alpha - a = 0 \tag{20}$$

Equations (13), (17), (18), and (20) implicitly determine optimal consumption as a function of the parameters of the model and the relative world prices π. The model is closed when prices π are determined. This is particularly simple in a symmetric setting.

3.1 The Symmetric Equilibrium

The model is particularly easy to solve in the symmetric case of identical countries. In that case, relative world prices are one ($\pi = 1$) and it is straight forward to derive from (18) and (20) the following

Proposition 3 *In a symmetric world economy the trade share e and the number of imported varieties $2\bar{r}$ are decreasing in trade costs T and δ, constant in labor L, and increasing in productivity a.*

This proposition states that, just as in standard models (Ricardian as in Dornbusch Fischer and Samuelson (1977), Heckscher-Ohlin, or monopolistic competition as in Helpman and Krugman (1985)), there is no scale effect in the trade share in the sense that multiplying the labor force L of every country in the world, the world trade share remains unaffected. Very much unless those standard models, however, the trade share increases the present model with labor productivity a. This relation reflects the core mechanism of the paper: higher labor productivity translates into higher wages and shifts out the individuals' budget set. As pointed out above, the decrease in the marginal sub-utility from imports is milder than that of the marginal sub-utility from domestic varieties. Consequently, the marginal propensity of expenditure on imports rises with an increasing income, driving up the trade share. Parallel to the rise in the trade share the origin-margin expands and the number of imported varieties of the representative country increases. This effect stems from the consumer's willingness to pay a higher price for foreign varieties as her total income increases. The basket of purchased foreign varieties thus grows with per capita income.

According to Proposition 3 per capita income drives up the trade share and the amount of imported varieties, defined as goods differentiated by origin. Thus, the general equilibrium model can qualitatively replicate the parallel growth of the trade share and the number of traded varieties per good shown in Figure 1. But imposing symmetry among countries was a cheap way to close the model by imposing constant terms of trade. A next step will analyze to what extend the Proposition 3 survives when departing from the strong assumption of symmetry and conducting comparative statics on the individual country parameters.

3.2 An Asymmetric Equilibrium

The number of imported varieties $2\bar{r}$ and the trade share e of country 0 are determined by equation (18) and (20). These variables differ from those of the mass of identical ROW-countries, which have to be calculated separately.

In order to do so, notice first that the deviation of country 0's characteristics implies that the ROW-countries are not identical anymore as they differ according to their location relative to country 0. This could in general introduce complicated heterogeneity among the ROW-countries. But as countries are atomistic, country 0's exports to any of the ROW-countries has zero weight in the respective import bundle so that the ROW-countries are still identical in two relevant variables: the trade share e^* and the maximal trade distance among each other r^*. Both variables can be determined for all ROW countries by replacing the terms of trade $1/\pi = 1$ in the equivalent of (18) and (20).

$$\alpha^* T^* \left(1 + \delta(\bar{r}^*)^\gamma\right) + (\bar{r}^*)^{\gamma+1} T^* \frac{2\gamma\delta}{\gamma+1} - \alpha^* - a = 0 \tag{21}$$

and

$$e^* = \frac{2\gamma}{\gamma+1} \delta T^* \frac{(\bar{r}^*)^{\gamma+1}}{a^*} \tag{22}$$

The maximal distance over which country 0's exports are shipped, however, differs from $\bar{r}^*$ and has to be determined separately. Let this distance be denoted by $\bar{\rho}$. Using the generic optimality condition (12), one calculates the relation between $\bar{r}^*$ and $\bar{\rho}$ to be

$$T^* p^* (1 + \delta(\bar{r}^*)^\gamma) = Tp(1 + \delta\bar{\rho}^\gamma) \tag{23}$$

whenever $\bar{r}^*$ and $\bar{\rho}$ are positive. Now notice that total demand for country 0's exports is $L^* 2 \int_0^{\bar{\rho}^*} T(1 + \delta r^\gamma) c^*_{r,o} dr$, while its exports are Lae. When country 0's export market clears one gets thus

$$L^* T \delta \frac{2\gamma}{1+\gamma} \bar{\rho}^{1+\gamma} = aeL$$

and with (18)

$$\bar{\rho}^{1+\gamma} = \pi\lambda\bar{r}^{1+\gamma} \tag{24}$$

where λ stands for the relative country size $\lambda = L/L^*$. Following modeling strategies of other multi-country analysis (e.g. Anderson and van Wincoop (2003)), a symmetric border cost are imposed here, i.e. goods shipped from ROW to country 0 and the other way round face the same border cost T.

Solving now for relative prices $\pi = p^*/p$ in (20) and combining (23) and (24) gives

$$T^2 \frac{\alpha(1 + \delta\bar{r}^\gamma) + \frac{2\gamma\delta}{1+\gamma}\bar{r}^{1+\gamma}}{a + \alpha} + \delta\bar{r}^\gamma T^{\frac{2+\gamma}{1+\gamma}} \lambda^{\frac{\gamma}{1+\gamma}} \left(\frac{\alpha(1 + \delta\bar{r}^\gamma) + \frac{2\gamma\delta}{1+\gamma}\bar{r}^{1+\gamma}}{a + \alpha} \right)^{1/(1+\gamma)} = M^* \tag{25}$$

where $M^* = T^*(1 + \delta(\bar{r}^*)^\gamma)$ is a constant in country 0's variables.

Finally, with π from (20) one can rearrange (18) to get

$$e = \frac{2\gamma}{1+\gamma} \frac{a+\alpha}{a} \frac{1}{\frac{2\gamma}{1+\gamma} + \alpha\bar{r}^{-1} + \alpha\bar{r}^{-(1+\gamma)}/\delta} \tag{26}$$

which shows that the trade share e is increasing in $\bar{r}$. Together, the identities (25) and (26) lead to the following

Proposition 4 *The number of varieties imported by country 0,* $2\bar{r}$*, is increasing in* a*,* a^**, and* T^* *and decreasing in* T *and* λ*. Country 0's trade share,* e*, is increasing in* a^**, and* T^* *and decreasing in* T *and* λ*.*

Proof. The left hand side of (25) is decreasing in a and increasing in T, λ, and $\bar{r}$. Further, M^* is constant in these variables and, as one can check with (21), increasing in T^* and a^*. This proves the first of the proposition. The left hand side of equation (26) only depends on $\bar{r}$, a and parameters of the model. As it is increasing in $\bar{r}$, this together with the first part of the proposition proves the second part. ■

Proposition 4 establishes an unambiguously positive relation between the number of varieties imported by a country and its per capita income. Just as in the symmetric equilibrium an individual country's trade share proves to increase in its productivity due to its increased inclination to buy imported goods, caused by rising income. Note that its terms of trade is moving against it, but the demand effect is stronger and overcompensates the adverse terms of trade effect.

The Proposition further points at the positive effect of the ROW-income a^* on country 0's trade share and the number of its imported varieties This reflects appreciation of country 0's terms of trade due to increased demand for its exports. Similarly, an increase in the relative size of ROW countries (a drop in λ) positively affects country 0's trade through this standard channel.

The relation between country 0's trade variables (e and $\bar{r}$) and the border cost between ROW countries, T^*, is positive. This positive relation is fairly intuitive: an increase in T^* makes between-ROW costlier and deviates ROW-demand for imports towards country 0 varieties. Consequently, country 0 experiences an appreciation of its terms of trade, which leads to and increase in its trade share and the number of its imported varieties. Conversely, an increase in the trade cost T reduces country 0's trade share and the number of its imported varieties.

While Proposition 4 shows that the number of varieties imported by country 0 is increasing in its productivity a, no statement has been made about a's effect on trade share e. In fact, as can be seen easily, an increase of a can have ambiguous effects on e. Consider the extreme case where country 0 is extremely unproductive and produces a negligible amount of its varieties while at the same time its rich ROW neighbors have a high demand for country 0's varieties ($a \to 0$ and $a^* \to \infty$). In that case, the equilibrium price of country 0's varieties exceeds the price of its imports. As in country 0 individuals consume the cheapest varieties only, its trade share then equals unity. Naturally, country 0's trade share must fall when it catches up with the ROW in terms of per capita income, and its trade share starts falling as individuals in country 0 begin to consume also the (relatively expensive) domestic varieties.

To rule out this extreme case one may want to impose that, in domestic prices, country 0's domestic varieties are less expensive than the varieties of its next neighbors at marginal output levels ($a \to 0$) or, using (19) that

$$T\pi \geq 1$$

is satisfied. This condition implies zero trade at very low productivity levels a, which is consistent with optimal consumption of country 0's variety by the ROW if $T^* p^* (1 + \delta(\overline{r}^*)^\gamma) < Tp$ holds (use (23) with $\bar{\rho} = 0$), or simply

$$\pi M^* \leq T.$$

Both conditions together give $T^2 > M^*$. It turns out that this latter condition is already sufficient for the trade share e to be increasing in productivity a and one can prove the following

Proposition 5 *If condition $T^2 > M^*$ holds, country 0's trade share e is increasing in its productivity a.*

Proof. See Appendix. ■

The Propositions 3 to 5 stress the role of productivity as a determinant of trade shares and the number of imported varieties per good. The present model thus suggests income per capita as a joint determinant explaining the dynamics reported in Figure 1 to 3 and gets the trends observed in the data qualitatively right. This success calls for a quantitative assessment of the effect, which will be performed further below. Before turning to the quantitative parts, however, a weak aspect of the model remains to be addressed. Figure 1a illustrates that not only the number of imported varieties per good but also the number of imported goods has grown considerably since the 1970ies and that this trend has paralleled the rise of trade volumes. The present paper has explicitly set out to provide an explanation for the rise in imported varieties *per* good but up to now the number of goods is arbitrarily fixed to a constant. However, a mechanism that explains the dynamics of a ratio by artificially setting the denominator to a constant does not deserve much credit.

To remedy this defect, the model will be extended to include a dimension of goods that will be traded endogenously. Compensating for the arising analytical complications of this extension the simplifying assumption of linear transport cost will me made at the same time.

3.3 Linear Transport Cost and a Continuum of Goods

In order to be able to address the increase in the number of imported goods documented in Figure 1a and elsewhere (see e.g. Broda and Weinstein (2004) and Kehoe and Ruhl (2002)), the current model is extended along a dimension of goods. To keep the model tractable, the transportation costs are assume to be linear in distance. This assumption means that the parameter γ is set to unity. Replacing a, the consumer's total expenditure by h, her expenditure on a specific good the linear transport cost allows the derivation of close form solutions for the central variables e and $\bar{r}$ as equations (18) and (20) can now be written as

$$e(h) = 1 + \frac{1}{h}\left(b - \alpha\sqrt{T\pi\delta}\sqrt{ha + c}\right) \tag{27}$$

$$\bar{r}(h) = \sqrt{\frac{h + c}{\delta T\pi}} - \alpha/2 \tag{28}$$

with the shorthand $b = \alpha + \alpha T\pi\,(\alpha\delta/2 - 1)$ and $c = \alpha + \alpha T\pi\,(\alpha\delta/4 - 1)$. These explicit expressions will prove useful in the following.

Assume now that there is a continuum of goods of total mass one. All goods are differentiated by domestic and foreign varieties as described above and the production of varieties takes place as in (6) for each variety and alike in all countries

$$x_{ikn} = a_i L_{ikn} \qquad\qquad n \in [0, 1] \quad k \in V_i \quad i \in C$$

where n is the good's index.

Just as the single good in the previous section, a good $n \in [0, 1]$ is either traded whenever condition (19) is satisfied on the goods level, i.e. when

$$\alpha(T\pi - 1) < h_n$$

holds, where h_n is the expenditure on the good considered. Clearly, a country with terms of trade π only imports varieties of goods that have an expenditure exceeding the threshold $h_o = \alpha(T\pi - 1)$. This threshold defines an endogenous partition of the set of goods into traded goods with high expenditure and nontraded goods with low expenditure.

According to these definitions, all goods are identical except for their expenditure share. The expenditure shares are assumed to be constant in prices and income - in other words goods enter consumers' utility according to a Cobb-Douglas aggregator. The distribution of expenditure can be described by a function $\varphi : (0, \infty) \to [0, \infty)$, where $\varphi(h)$ determines the mass of goods with expenditure $h > 0$. Since the total mass of goods is normalized, $\varphi(h)$ integrates to one and represents a density function. With the help of the function $\varphi(h)$ the total number of imported goods, total expenditure on foreign goods, and the total number of imported varieties, of a country can be written, respectively, as

$$
\begin{aligned}
G &= \int_{\alpha(T\pi-1)}^{\infty} \varphi(h)\,dh \\
E &= \int_{\alpha(T\pi-1)}^{\infty} he(h)\varphi(h)\,dh \\
R &= \int_{\alpha(T\pi-1)}^{\infty} 2\bar{r}(h)\varphi(h)\,dh
\end{aligned}
\tag{29}
$$

where $e(\,.\,)$ and $\bar{r}(\,.\,)$ are the functions from (27) and (28).

A closer look at these expressions reveals that with an adequate choice of the density φ it is possible to generate almost arbitrary responses of each of the trade volume E (or G or R) to increases in per capita income[5]. Thus, φ must be carefully chosen in accordance to the data. It will turn out that the following truncated Power-distribution of φ captures a strong regularity in the data

$$
\varphi(h) = \begin{cases} D{\cdot}h^{-\theta} & if \quad h \in [\bar{h}, d\bar{h}] \\ 0 & else \end{cases} \tag{30}
$$

where $d \in (1, \infty)$. The normalization of the mass of goods ($\int \varphi(h)dh = 1$) and the requirement that total expenditure equals income ($\int h\varphi(h)dh = a$) determine the constants

$$
D = \frac{1-\theta}{1-d^{1-\theta}}\bar{h}^{\theta-1} \qquad \text{and} \qquad \bar{h} = \frac{\theta-2}{\theta-1}\frac{1-d^{1-\theta}}{1-d^{2-\theta}}a
$$

while the parameters θ and d govern the shape of the distribution φ. Note that there is a linear relationship between the $\bar{h}$ and a which implies that comparative statics with respect to a can be performed by varying the parameter $\bar{h}$ while holding θ and d constant.

In the following, consider again the symmetric equilibrium characterized by $\pi = 1$. Under the assumption that not all goods are traded, i.e. when

$$
\bar{h} < \alpha(T-1) < d\bar{h} \tag{31}
$$

holds, one checks with definitions (30) that the trade share and number of goods and varieties are increasing in world per capita income. In particular, one can prove the following

Proposition 6 *Assume (31) holds. In a symmetric world economy the trade share, E/a, and the number of varieties, R, are decreasing in trade costs, T and δ, constant*

[5] In that sense, the funcion φ is the equivalent to the A-line of comparative advantage in Dornbusch Fischer and Samuelson (1977), the choice of which implies arbitrarily large responses to reductions off trade costs.

in labor, L, and increasing in world per capita income, a. The number of goods, G, is decreasing in T, constant in δ and L, and increasing in a. Moreover, the average number of varieties per good, R/G, is increasing in world per capita income a.

Proof. Using equations (27) - (29), the statements concerning E/a, G, and R are immediate. The ratio R/G is increasing in $\bar{h}$ (and therefore in a) since

$$\frac{d}{d\bar{h}}\frac{R}{G} = d\left\{\frac{2\bar{r}(d\bar{h})}{R} - \frac{1}{G}\right\}\frac{R}{G}\varphi(d\bar{h}) > 0$$

and $R < \int 2\bar{r}(d\bar{h})\varphi(h)\,dh = 2\bar{r}(d\bar{h})\cdot G$. ■

The proposition states that an increase in absolute expenditure in all good categories makes some of the previously nontraded goods jump the critical level of expenditure (19) and become traded goods, increasing the number traded goods. Moreover, the number of traded varieties increases because for every traded good, the number of traded varieties increases (compare equation (28)) and on top of that, new good with new varieties are traded. By these two forces and the intensive margin, the total trade share also increases.

The rise in the number of traded varieties per good, R/G, in response to an increase in a is the less intuitive result. In fact, there is one force that tends to lead to a reduction of the average number of varieties per good: the number of goods traded at negligible amounts has the mass $\varphi(\alpha(T-1)/a)$, which is larger than any traded good. Now, an incremental increase in a adds these goods with a large mass and a low number of varieties per good to the traded basket, which tends to reduce the average R. To see that this effect does not dominate the ratio R/G, take two expenditure shares h_1 and h_2 and observe that the *relative* mass of goods with the respective expenditure is constant in $\bar{h}$. Thus, the relative weights on the values $\bar{r}(h)$ do not change - except that at the upper end of the distribution the mass of goods $\varphi(d\bar{h})$ with the maximal expenditure $d\bar{h}$ are added. This margin a the upper end in fact creates the rise in the ration R/G, the average number of varieties per good.

With the model augmented by the dimension of goods with an endogenously determined partition into traded or non-traded goods and a non-trivial extensive margin, the calibration will be performed in next section. In addition, the key implication regarding per capita income is tested in order to further assess the performance the model's prediction.

4 Calibration and the Gravity Equation

The aim of this section is to bring the model to the data and evaluate its performance quantitatively. With the help of a simple calibration exercise the first part shows that the dynamics of the US trade share and the number of imported goods and varieties can be mimicked quite well. With a log-linear approximation of the basic model, a second part derives a version of the gravity equation, augmented by per capita income. In a consecutive test, the key predictions regarding the role of per capita income cannot be rejected.

4.1 A Calibration exercise

In the following exercise, the model is calibrated to match the time-series of US trade share and the number of imported goods and varieties using the free parameters are α, d, and the output units. Following Yi (2003), time series for T and δ are taken from US trade data. Similarly, the value θ is inferred from US data. Expenditure growth is approximated by US per capita output growth and, finally, symmetry is imposed on the model. This latter assumption of symmetric clearly constitutes a strong simplification, but as the growth performance of the US and the rest of the world is reasonably parallel over the period considered here (see Figure 5) the resulting error should be moderate.

The Parameter θ of the Distribution φ. In the present model the distribution of expenditure shares across goods has important consequences for the relation between income growth and trade share and the number of traded goods and varieties. The function φ must therefore be carefully chosen in accordance with the relevant data. Figure 1 has been generated with US trade data disaggregated by 22,000 goods categories classified by the "Harmonized System" (see Feenstra et al (2002)). Ideally, φ is calibrated according to the expenditure shares of these goods, but unfortunately data of US expenditure shares at these disaggregation levels are not available. One way to bypass this problem is to infer the underlying expenditure structure from trade data . To this goal, observe with (27) that $e(h) \to 1$ as h grows large. Making use of this observation, the nature of φ can be read from the upper end of the distribution of the expenditure shares $e(h)$. Figure 6 plots the nominal import volumes of the HTS-classification ordered by descending size[6] for the years 1972 and 2001 on a log-log scale. Apart from an upward shift of the schedule for 2001 (due to real import growth and inflation) the two curves have a similar shape and are very well approximated by a straight line with slope -1 (the solid line on top) for left part of the distribution, i.e. the goods with the larger expenditure[7]. Thus, assuming that h_n is large such that $e(h_n)$ is close enough at unity at this upper end of the distribution, the relation between the rank κ and expenditure h can inferred to satisfy approximately $\kappa \sim h^{-1}$. Consequently, the probability that the expenditure of a good, which is randomly chosen out of the pool of tradable

[6] The order is different for the two years.

[7] A regression of the log expenditure on log rank and a constant of the 5% traded categories with highest expenditure a coefficient of -0.93 (-.92) with a t-value of -397.71 (-196.85) and an R^2 of 0.99 (0.99) for the year 2000 (1972). This shows thatthe approximation is not perfect but reasonably good.

exceeds $\bar{h}$ is proportional to $\bar{h}^{-1}$ ($P(e(h) > \bar{h}) \sim \bar{h}^{-1}$). This implies a parameter of $\theta = 2$ in the density function (30).

With this parameter choice, one can use (28) and (27) to calculate the variables defined in (29). When condition (31) is satisfied the total number of imported varieties becomes[8]

$$R = D\left\{-\frac{1}{\sqrt{T\pi\delta}}\left[\frac{1}{\sqrt{c}}\tanh^{-1}\left(\sqrt{\frac{h}{c}+1}\right)+\frac{\sqrt{h+c}}{h}\right]+\frac{\alpha}{2h}\right\}_{\alpha(T-1)}^{d\bar{h}} \tag{32}$$

while total expenditure on tradables is

$$E = D\left\{\ln(h)-\frac{b}{h}+\alpha\sqrt{T\pi\delta}\left[\frac{1}{\sqrt{c}}\tanh^{-1}\left(\sqrt{\frac{h}{c}+1}\right)+\frac{\sqrt{h+c}}{h}\right]\right\}_{\alpha(T-1)}^{d\bar{h}} \tag{33}$$

(Remember $b = \alpha + T\pi\alpha\,(\alpha\delta/2 - 1)$ and $c = \alpha + T\pi\alpha\,(\alpha\delta/4 - 1)$. Finally, the number of imported goods is

$$G = D\cdot\left[1/(\alpha(T-1)) - 1/(d\bar{h})\right] \tag{34}$$

Trade costs T and δ. In an recent article Anderson and van Wincoop (2004) give an estimation and a decomposition of broadly defined trade costs. The authors estimate the border cost for industrialized countries to amount to a tariff equivalent of about 44% , less that 5% of which is due to tariffs. To these roughly 40% border cost ($T = 1.4$) the time series of tariff derived from data of Feenstra et al (2002) are added.

A standard measure for the raw border-to-border transportation costs is the cif over fob ratio (see e.g. Baier and Bergstrand (1999)) which can be derived from IMF data. However, this measure does not coincide with the per unit transportation costs δ in the present model. In fact, using (10) and (13) the cif/fob measure, when derived from one good only, depends on the parameter δ in the following way

$$\frac{cif_n}{fob_n} = \frac{\int T(1+\delta r)c_r\,dr}{\int Tc_r\,dr} = \frac{(\delta\bar{r})^2}{2\left((1+\delta\bar{r})\ln(1+\delta\bar{r})-\delta\bar{r}\right)}$$

which, with (28), is a function of the parameter of the model and can be shown[9] to be decreasing in T, constant in L, increasing in a and increasing and concave in

[8] Use $\int\frac{\sqrt{x+c}}{x^2}dx = -\frac{\sqrt{x+c}}{x} - \frac{1}{\sqrt{c}}\tanh^{-1}(\sqrt{x/c+1})$.

[9] Use Proposition 1 and $d\ln(cif/fob)/d\xi = -(\xi-\ln(1+\xi))/(\xi((1+\xi)\ln(1+\xi)-\xi))$ where $\xi = \delta\bar{r}$ for the statement on T, L, and a.

δ. In particular, the relation between the cif/fob measure and the "true" δ is a nonlinear one. The cif/fob measure derived from all imports, is

$$\frac{cif}{fob} = \frac{\int_{\alpha(T-1)}^{d\bar{h}} (\delta\bar{r})^2 \varphi(h)\, dh}{\int_{\alpha(T-1)}^{d\bar{h}} 2\left((1+\delta\bar{r})\ln(1+\delta\bar{r}) - \delta\bar{r}\right)\varphi(h)\, dh} \tag{35}$$

For fix T, a, and α and for a given data point of cif/fob, this identity determines implicitly the value of δ.

Figure 7 shows for the case for parameter values as in the "full" specification below. There is a highly non-linear and concave relation between the cif/fob measure and parameter δ - this means that the standard cif/fob measure of transportation cost is biased upwards at low values of δ and biased downwards for high values. As a direct consequence, the measured response in the cif/fob ratio to a change in the per unit transport cost δ differs strongly according to the level of δ. In addition to this bias, border costs T and per capita income a enter the cif/fob measure. These systematic measurement problems may be worth to analyze in detail. In particular, an evaluation of their role in empirical studies like Baier and Bergstrand (1999) seems appropriate. But such an analysis is not within the scope of the present paper and will not be further pursued here.

Instead, in the present calibration the measured cif/fob ratio is taken from the data, scaled up to match the value of 0.12 in the year 2000 measured by Anderson and van Wincoop (2004) and then used to numerically infer the parameter δ from (35) for given per capita income, α and border costs T.

Import Elasticity. The rise in trade volumes is considered to be a puzzle since standard models fail to explain it under realistic import elasticities. Any attempt to quantitatively address the issue must therefore consider the implied price elasticity of imports. Yi (2003) argues that the interval [2,3] is a reasonable range for import elasticities, these values will be the reference in the calibration.

In the framework of the present paper's model, the important elasticity deserves special attention since for each single variety the price elasticity is endogenous and ranges in the open interval $(1, \infty)$. It is thus neither constant nor bounded[10]. It is easy to see that a varieties' price elasticity changes with quantity and thereby depend, among others, on the exporter's distance r and thus on their origin. Moreover, the elasticities of goods, as an aggregate of the varieties, generally changes with the expenditure level such that there is no unique definition of an import elasticity in this model.

Before the calibration, a careful definition of import elasticity is therefore needed. A relatively simple and natural candidate for the definition is the elasticity derived

[10] The demand elasticity is $-(\ln(Q))'/(\ln(P)')$ and grows unbounded as price Q approaches the level at which demand C drops to zero.

from a uniform change in terms of trade the highest aggregation level. This gives the definition

$$\mathcal{E} = -\frac{\partial_\pi \ln(E)}{\partial_\pi \ln(Q)} \tag{36}$$

where $Q = E/C$ is the ideal price index of imports, E is expenditure on imports from (33),and C is the summation of all units of imported varieties. To derive the elasticity, Q and C are to be calculated. The total imported quantity is

$$C = 2\int_{\alpha(T-1)}^{d\bar{h}} \int_0^{\bar{r}} c_r \, dr d\varphi(h) = 2\int_{\alpha(T-1)}^{d\bar{h}} \int_0^{\bar{r}} \frac{\delta(r-\bar{r})}{1+\delta r} dr d\varphi(h)$$

The expression for the import elasticity is rather cumbersome and is presented in the appendix. The values for $\mathcal{E}$ are reported here along with the other calibration results.

Calibration Targets. The three free parameters for a calibration the model are (α, d, m). α is the mass of home-produced varieties, d governs the truncation of the distribution φ, and m represent the price of consumption units in 1996 dollars. These parameters are jointly used to target the US trade volume in 1972 and 2000 and the import elasticity. Further, as the number of goods and varieties imported are represented by a continuum in the model, the actual numbers of the variables are normed to match the data at the start of the period, i.e. in the year 1972.

Calibration Results. In the *basic calibration* all trade costs are fixed at their 1972 values $(T, cfi/fob) = (1.45, 0.12)$ and only US per capita income growth is fed into in the model. Figure 8a illustrates the results for the calibration with $(\alpha, d, m) = (20.1, 2750, 10^6)$. The top panel shows that the trade share predicted by the model replicates the dynamics of the data very well and parallels the swings of the time series. Further, the middle and the bottom panel show model's fit regarding the number of goods and the number of varieties. While the latter is rather on the higher side towards the end of the period considered, the general fit is quite satisfactory. It is worth remembering that the free parameters are used to match the trade shares and import elasticity such that the successful fit of the growth in the number of goods and varieties is not result of calibration these time series. The import elasticity, however, rages between 3.1 and 4.5, which is less than some empirical estimates the literature provides estimates (Baier and Bergstrand (1999) estimate an elasticity around 6) but above the target interval [2,3].

The *full specification* now incorporates the time series of tariffs and the cif/fob measure. Figure 8b graphically reports the calibration results with $(\alpha, d, m) = (18.4, 5.7 \cdot 10^6, 10^9)$ regarding the trade share, and the number of imported goods and varieties. Again, the model is very successful in calibrating the trade share and the number of imported goods while it overstates the number of imported varieties. The import elasticity, on the other hand, drops considerably in the full specification and ranges now between 2.4 and 3.05, which, by and large, can be considered as a successful calibration to the targeted range of [2,3].

With the parameters thus calibrated one can disentangle the effects of, respectively, the per capita growth, the tariff and the trade cost reduction on the trade share. This is done by freezing two of the three parameters to their 1972 level and feeding only the data of the third into the model. This exercise shows that the model attributes 4.05 percentage points (or 60.5 % of the total) of the rise in the trade share to the increase in per capita income, 2.05 percentage points (or 30.6 % of the total) to the fall in transport cost, and only 0.33 percentage points (or 4.9 % of the total) to tariff reductions. This latter result is particularly noteworthy as it starkly contrasts the finding of Baier and Bergstrand (2004) who estimate in the framework of the gravity equation that the impact of tariff reduction by far dominates the impact of the reduction in transport cost. This calibration most obviously cannot replace a full econometric analysis, but the above findings hint at the possibility that Baier and Bergstrand's (2004) results may be sensitive to the introduction of per capita income in their estimation and to a more careful treatment of the transportation cost.

Necessary for a rigorous evaluation is the introduction of these changes into the standard empirical framework used to assess bilateral trave volumes, the gravity equation. A fist step in that direction will be done next.

4.2 Towards the Gravity Equation

Anderson and van Wincoop (2003) describe the gravity equation as "one of the most empirically successful in economics". It relates the bilateral trade volume of any pair of countries to trade costs and their respective size and it represents an authoritative framework to test theories regarding trade flows. Empirically successful international trade models must, as a minimum requirement, be consistent with the gravity equation, and, if possible, give testable predictions in its context. The next paragraphs derive a version of the gravity equation from the model developed above. In order to do so, return to the stylized and tractable setup with one single good and a continuum of varieties produced by the different countries. Further, the assumption of linear transport cost ($\gamma = 1$) will be kept.

Up to this stage, all countries but a single one (country 0) were assumed to be identical. This assumption made the model tractable and allowed a convenient formulation of its key variables. In particular, taking country 0's terms of trade ($1/\pi$) as given, its trade share was described by equations (27) and (28) and generally differed from that of the homogeneous rest of the world countries. In order to derive meaningful predictions concerning bilateral trade flows, however, the model must allow for more variation in the country parameters. In particular, the country characteristics of two different countries (exporter and importer) must be able to deviate from the average.

In order to enrich the model in that respect the previous setup is mildly generalized by introducing a subset Z of countries with zero measure that differ from the otherwise identical ROW. This generalization allows exporter and importer prices to vary from the average while at the same time the equations that led to (21) - (24) still go through unchanged for every country $i \in Z$, where prices p_i, productivity a_i, and population L_i have to be marked the country index $i \in Z$ now.

With the additional restriction $\gamma = 1$ the relevant equations (23) and (24) now become

$$T^*(1+\delta\bar{r}^*)\pi_i = T_i(1+\delta\bar{\rho}_i) \qquad \text{and} \qquad \bar{\rho}_i^2 = \pi_i\lambda_i\bar{r}_i^2$$

where T_i represents the symmetric border cost between country i and the ROW and distances $\bar{\rho}_i$ and $\bar{r}_i$ are those of country i relative to the ROW (i.e. $\bar{r}_i = \sup_k\{\, r_{ik} \mid c_{ik} > 0 \quad k \in C\backslash Z\,\}$). These two equations can be combined to

$$T_i/\pi_i + T_i\delta\sqrt{\lambda_i/\pi_i}\bar{r}_i - T^*(1+\delta\bar{r}^*) = 0 \tag{37}$$

and $\bar{r}_i$ is determined by (compare (28))

$$T_i\pi_i\left(\delta\bar{r}_i^2 + \delta\alpha\bar{r}_i + \alpha\right) - a - \alpha = 0 \tag{38}$$

Finally, the value of imports of a country $i \in Z$ from any other country $j \in C\backslash\{i\}$, measured in ROW-prices will be labeled B_{ij}. To determine B_{ij}, one can use the generic optimality condition (12) to get

$$B_{ij} = L_i q_{ij} c_{ij} = L_i(T_i\left(1+\delta\bar{r}_i\right) - T_{ij}p_j(1+\delta r_{ij})) \tag{39}$$

Remember that q_{ij} and p_i are the respective consumer and producer prices as defined in (10). To estimate equation (39) with standard techniques, one can (log-) linearize this equation around the average world variables for the variables a_i, a_j, L_i, and L_j and around $\bar{r}/2$ for r.

In a first step use (38) to get

$$\begin{aligned}
\frac{\partial\bar{r}_i}{\partial\pi_i} &= -\frac{1}{\pi_i}\frac{\bar{r}_i^2+\alpha\bar{r}_i+\alpha/\delta}{2\bar{r}_i+\alpha}\\
\frac{\partial\bar{r}_i}{\partial T_i} &= -\frac{1}{T_i}\frac{\bar{r}_i^2+\alpha\bar{r}_i+\alpha/\delta}{2\bar{r}_i+\alpha}\\
\frac{\partial\bar{r}_i}{\partial a_i} &= \frac{1}{\delta T_i\pi_i}\frac{1}{2\bar{r}_i+\alpha}
\end{aligned}$$

This gives with (37) the derivatives of π_i w.r.t. a_i, λ_i, and T_i around the symmetric equilibrium characterized by $a_i = \lambda_i = 1$ and $\bar{r}/2$, this is

$$\begin{aligned}
\frac{d\pi_i}{da_i} &= \frac{2}{T}\frac{1}{4\bar{r}(\delta\bar{r}+1)+\alpha(3\delta\bar{r}+4)}\\
\frac{d\pi_i}{d\lambda_i} &= \frac{\delta\bar{r}(2\bar{r}+\alpha)}{4\bar{r}(\delta\bar{r}+1)+\alpha(3\delta\bar{r}+4)}\\
\frac{d\pi_i}{dT_i} &= \frac{2\bar{r}}{T}\frac{\delta\bar{r}+2}{4\bar{r}(\delta\bar{r}+1)+\alpha(3\delta\bar{r}+4)}
\end{aligned} \tag{40}$$

With the help of these derivatives one gets the log-linearization (39) around the symmetric equilibrium as

$$\widehat{\ln(B_{ij})} \approx \theta_a \left(\widehat{\ln(a_i)} + \widehat{\ln(a_j)}\right) + \theta_L \left(\widehat{\ln(L_i)} + \widehat{\ln(L_j)}\right) + \theta_T \widehat{\ln(T_{ij})} - \widehat{\ln(r_{ij})} \quad (41)$$

with the coefficients

$$\begin{aligned}
\theta_a &= \frac{2}{T\delta\bar{r}} \frac{\delta\bar{r} + 2}{4\bar{r}(\delta\bar{r} + 1) + \alpha(3\delta\bar{r} + 4)} a \\
\theta_L &= \frac{(2\bar{r} + \alpha)(\delta\bar{r} + 2)}{4\bar{r}(\delta\bar{r} + 1) + \alpha(3\delta\bar{r} + 4)} \\
\theta_T &= -\frac{\delta\bar{r} + 2}{\delta\bar{r}}
\end{aligned}$$

The hat in equation (41) stand for the deviation from the ROW-variables ($\widehat{\ln(X_{ij})} = \ln(X_{ij}) - \ln(X_{ROW})$).

The linearized model suggests with equation (41) a testable version of the gravity equation augmented by productivities a_i. For practical reasons, however, it is convenient to rewrite (41) in variables that are available in the data and typically used in trade regressions, i.e. GDP and GDP per capita. To do so, consider changes in GDP due to the size of the labor force only and note that $\ln\widehat{(GDP_k)} = \ln\widehat{(p_k a L_k)} = \widehat{\ln(p_k)} + \widehat{\ln(L_k)}$. Use further $p_k = 1/\pi_k$ and (40) to confirm

$$\widehat{\ln(p_k)} = -\frac{\delta\bar{r}(2\bar{r} + \alpha)}{4\bar{r}(\delta\bar{r} + 1) + \alpha(3\delta\bar{r} + 4)} \widehat{\ln(L_k)}$$

such that

$$\theta_L \widehat{\ln(L_k)} \approx \frac{\bar{r} + \alpha/2}{\bar{r} + \alpha} \ln\widehat{(GDP_k)}$$

Parallel calculations for the variation in a_k lead to the identity

$$\theta_a \widehat{\ln(L_k)} = \frac{\delta\bar{r} + 2}{\delta\bar{r}} \frac{T(\delta\bar{r}^2 + \alpha\delta\bar{r} + \alpha) - \alpha}{T(\delta\bar{r}^2 + (2 + \alpha\delta/2)\bar{r} + \alpha) + \alpha} \ln\widehat{(gdp_k)}$$

where $gdp_k = p_k a_k$ stands for per capita GDP in country k. As total GDP picks up any variation in per capita GDP, a econometric test will estimate the following equation[11]

$$\widehat{\ln(B_{ij})} = \theta_1 \left(\ln\widehat{(GDP_i)} + \ln\widehat{(GDP_j)}\right) + (\theta_2 - \theta_1) \left(\widehat{\ln(gdp_i)} + \ln\widehat{(gdp_j)}\right) - \widehat{\ln(r_{ij})} \quad (42)$$

[11] In (41) and te following test the variation in border costs T is suppressed for simplicity.

with the coefficients

$$\theta_1 = \frac{\bar{r} + \alpha/2}{\bar{r} + \alpha}$$
$$\theta_2 = \frac{\delta\bar{r} + 2}{\delta\bar{r}} \frac{T(\delta\bar{r}^2 + \alpha\delta\bar{r} + \alpha) - \alpha}{T(\delta\bar{r}^2 + (2 + \alpha\delta/2)\bar{r} + \alpha) + \alpha}$$

It is quick to check that the parameters are within the following range $\theta_1 \in (1/2, 1)$ and $\theta_2 \geq 1$ such that $\theta_2 - \theta_1 \geq 0$.

Thus, the econometric model derived from the theory is

$$\widehat{\ln(B_{ij})} = \beta_0 + \beta_1 \left(\widehat{\ln(gdp_i)} + \widehat{\ln(gdp_j)}\right) ... \\ ... + \beta_2 \left(\ln(\widehat{GDP_i}) + \ln(\widehat{GDP_j})\right) - \beta_3 \widehat{\ln(r_{ij})} + \varepsilon_{ij} \tag{43}$$

where ε_{ij} is interpreted as a measurement error. Equation (43) is estimated with standard OLS using data of nominal bilateral trade volumes on a yearly basis between 1962 and 2000 for 150 countries (for a description of the data see Feenstra et al (2005)). This gives about a quarter of a million observations. There are six specifications of (43) reported below, that differ slightly from each other. All estimations include time dummies.

Estimation Results

When reviewing the regression results two predictions on the coefficients β_i differ from the standard models and therefore deserve particular attention. These predictions are, first, that the impact of per capita income is positive and between one half and unity ($\beta_1 \in (0.5, 1)$), second, that the coefficient on total GDP is positive but may be smaller than unity ($\beta_2 > 0$). A third prediction concerns the parameter on distance, which equals one $\beta_3 = 1$. It is noteworthy that this prediction is independent of the actual transport cost δ. For comparison the "traditional" gravity equation, i.e.

$$\ln(B_{ij}) = (\ln(GDP_i) + \ln(GDP_j)) - \ln(r_{ij})$$

are also reported.

Table 3 reports the estimates of equation (43). Compared with that traditional gravity equation (column 1) the inclusion of the per capita incomes (column 3) raises the R^2 from 0.53 to 0.59. Concerning the actual estimates, observe first that the estimated coefficients on log per capita incomes are significant on all conventional levels and, as predicted, larger than one half and smaller than unity. Second, the estimated coefficients on total national incomes drop significantly when introducing per capita income and are much lower than unity. This contradicts the traditional gravity equation but is consistent with the predictions of the current model. Finally, and as usual in the gravity regressions, the estimate of the coefficient on distance is close to unity and the null that it is one cannot be rejected on the 5% confidence level in specifications 4 and 6. Moreover, columns 5 and 6 report estimations of the augmented gravity equation including per capita income and total GDP of the source country, showing that the estimates of the individual country variables are very close to each other. In fact, the estimated coefficient of per capita income of the source country can only be statistically distinguished from zero if landlocked-dummies are included.

This paper started out with the aim to jointly explain the rise in US trade volumes and the number of traded goods and varieties. The motivation was further based on the observation by Haveman and Hummels (1999) that countries purchase only a small fraction of varieties available on the world market. The model developed here could account for this observation by introducing a mild modification of standard consumer preferences, which gave rise to some novel results. Alternative approaches to address this observation can be based on the now popular assumptions of fixed cost to enter export markets (see e.g. Melitz (2003)), which generally implies that strict subsets of available varieties are purchased and consumed in each country. In this class of models, however, the set of imported varieties is determined by trade costs and the size of the export market (i.e. the importer's GDP); per capita income becomes entirely inessential (see Chaney (2005)). The fact that the coefficient on the per capita income of the importing country is estimated to be positive and highly significant (in both, statistical and economic terms) can be viewed as supportive to the approach pursued in the present paper.

5 Conclusion

The present paper has addressed the substantial and parallel growth of trade shares and the *origin-margin*, i.e. the number of source countries per imported good. Increases in per capita income has been suggested as a joint determinant for the parallel trends. The mechanism presented is based on the assumption that varieties are nonessential in the consumers' utility function. Under ex-ante identical preferences, rich consumers are more willing to incur high transport costs and purchase varieties from more distant locations, such that the set of imported varieties grows with per capita income. This origin-margin of imports implies that the marginal utility of imports exhibits milder decreasing returns than the marginal utility of domestic goods. Through this channel, the growth in per capita income raises the trade share of a country. Moreover, aggregate trade volumes and imported varieties per good are linked through per capita income and move together. Testing the effects of per capita income on the bilateral trade flows within the framework of the gravity equation provided support for the derived hypothesis. Finally, the calibrated model is able to essentially replicate the dynamics of US imports, and the number of imported goods and varieties between 1972 and 2000.

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A Appendix

The Import Elasticity The ideal price index for the bundle of imported goods is

$$Q = E/C \tag{A1}$$

where E is country 0's expenditure on imports (33) and C is the total sum of imported quantities. The price change defining the price elasticity of imports is assumed to stem from a terms of trade shock that is uniform across exporting countries. When $d/d\pi$ is denoted by a prime of variables in question, this means that the price elasticity of imports is $\mathcal{E} = -(\ln(C))'/(\ln(Q))'$ or

$$\mathcal{E} = \frac{\ln(C)'}{\ln(C)' - \ln(E)'}$$

Using (33), derivatives of E are quickly calculated and

$$\frac{1}{D}\frac{dE}{d\pi} = \left[-\frac{4T}{h} + \sqrt{T}\left\{ \tanh^{-1}(\sqrt{h/4+1}) + 2\frac{\sqrt{h+4}}{h} \right\} \right]_{4(T-1)}^{\bar{h}} \ldots$$

$$\ldots - 4T\left\{ \ln(4(T-1)) + 1 + 2\sqrt{T}\tanh^{-1}(\sqrt{T}) \right\}$$

Now define the quantity of total imports as the sum over all goods and varieties:

$$C = 2\int_{4(T-1)}^{\bar{h}} \int_{0}^{\bar{r}(h)} c(r,h)\,dr\,d\varphi(h) \tag{A3}$$

With the expression of $\bar{r}$ from (28), one can take the derivative of C with respect to import prices π at the symmetric equilibrium ($\pi = 1$)

$$\frac{dC}{d\pi} = 2\int_{4(T-1)}^{\bar{h}} \int_{0}^{\bar{r}(h)} \frac{dc(r,h)}{d\bar{r}}\frac{d\bar{r}}{d\pi}\varphi(h)\,dr\,d\varphi(h) \ldots$$

$$\ldots + 2\int_{4(T-1)}^{\bar{h}} c(\bar{r},h)\frac{d\bar{r}}{d\pi}\,d\varphi(h) - 4T\int_{0}^{\bar{r}(4(T-1))} c(r,h)\,dr\,\varphi(4(T-1))$$

The two last terms equal zero such that, with $d\bar{r}/d\pi$ from (28) and $\varphi(h)$ from (30)

$$\frac{dC}{d\pi} = D\int_{4(T-1)}^{\bar{h}} \frac{\sqrt{(h+4)/T}}{h^2}\ln\left(\sqrt{(h+4)/T} - 1\right)dh \tag{A4}$$

Now, use (12) and (28) to get the import quantity (A3) for the symmetric equilibrium

$$C = 2D \int_{4(T-1)}^{\bar{h}} \frac{1}{h^2} \left[(1+\bar{r})\ln(1+\bar{r}) - \bar{r}\right] dh$$

$$= 2D \int_{4(T-1)}^{\bar{h}} \frac{1}{h^2} \left[\left(\sqrt{\tfrac{h+4}{T}} - 1\right) \ln\left(\sqrt{\tfrac{h+4}{T}} - 1\right) - \sqrt{\frac{h+4}{T}} + 2\right] dh$$

$$= 2\frac{dC}{d\pi} - 2D \int_{4(T-1)}^{\bar{h}} \frac{\ln\left(\sqrt{\frac{h+4}{T}} - 1\right)}{h^2} dh - 2D \int_{4(T-1)}^{\bar{h}} \frac{1}{h^2}\sqrt{\frac{h+4}{T}} dh - 4D \left[\frac{1}{h}\right]_{4(T-1)}^{\bar{h}}$$

The two integrals can be solved and the whole expression becomes

$$C = 2\frac{dC}{d\pi} - 4D\left[\frac{1}{h}\right]_{4(T-1)}^{\bar{h}} ...$$

$$-2D\frac{1}{4-T}\left[\frac{\sqrt{T}}{2}\tanh^{-1}\left(\sqrt{\tfrac{h}{4}+1}\right) - \frac{1}{2}\ln(h) \;...\right.$$

$$\left. ... + \ln(\sqrt{h+4} - \sqrt{T}) - \frac{4-T}{h}\ln\left(\sqrt{\frac{h+4}{T}} - 1\right)\right]_{4(T-1)}^{\bar{h}}$$

$$... + 2D\left[\frac{\sqrt{h+4}}{h} + \frac{1}{2}\tanh^{-1}(\sqrt{h/4+1})\right]_{4(T-1)}^{\bar{h}}$$

Unfortunately, there is no close form solution for the derivative $dC/d\pi$ in (A4). It can, however be written as

$$\frac{dC}{d\pi} = 2D\left\{\left(\ln\left(\sqrt{\tfrac{h+4}{T}} - 1\right) - 1\right)\left(\sqrt{\tfrac{h+4}{T}} - 1\right) ...\right.$$

$$... + \ln\left(\sqrt{\tfrac{h+4}{T}} - 1\right) 2\left(\tanh^{-1}(\sqrt{h/4+1}) - \tanh^{-1}(\sqrt{T}/2)\right) ...$$

$$\left. ... + \mathrm{Li}_2\left(\frac{\sqrt{h+4} - \sqrt{T}}{\sqrt{T} - 2}\right) - \mathrm{Li}_2\left(\frac{-\sqrt{h+4} + \sqrt{T}}{\sqrt{T} + 2}\right)\right\}_{4(T-1)}^{\bar{h}}$$

where Li_n is the Polylogarithmic function $\mathrm{Li}_n(x) = \sum_{k=1}^{\infty} z^k/k^n$. Combining finally E, $dE/d\pi$, C, and $dC/d\pi$ gives the price elasticity of imports $\mathcal{E}$.

Proof of Proposition 5. Show: for $T^2/M^* \geq 1$ the trade share e of country 0 is increasing in a. Use (23) and (24) to write (20) as

$$a = T\left[\frac{2\gamma}{\gamma+1}\frac{\delta}{\lambda}\bar{\rho}^{\gamma+1} + \frac{\alpha\delta}{\lambda^{\gamma/(\gamma+1)}}\bar{\rho}^{\gamma}\left(\frac{T}{M}(1+\delta\bar{\rho}^{\gamma})\right)^{\frac{1}{\gamma+1}} + \alpha\delta\bar{\rho}^{\gamma}\frac{T}{M^*}\right] + \alpha\left(\frac{T^2}{M^*} - 1\right)$$

As the left hand side is increasing in $\bar{\rho}$, this means that $\bar{\rho}$ is increasing in a. Further, one gets

$$\frac{\bar{\rho}^{\gamma+1}}{a} = \left\{T\left[\frac{2\gamma}{\gamma+1}\frac{\delta}{\lambda} + \frac{\alpha\delta}{\lambda^{\frac{\gamma}{\gamma+1}}}\left(\frac{\frac{T}{M}(1+\delta\bar{\rho}^{\gamma})}{\bar{\rho}^{\gamma+1}}\right)^{\frac{1}{\gamma+1}} + \frac{\alpha\delta}{\bar{\rho}}\frac{T}{M^*}\right] + \frac{\alpha}{\bar{\rho}^{\gamma+1}}\left(\frac{T^2}{M^*} - 1\right)\right\}^{-1}$$

Since $T^2 \geq M^*$, the expression on the left hand side is increasing in $\bar{\rho}$ and therefore in a. Rewriting finally (18) with (24) as

$$e = \frac{2\gamma}{\gamma+1}\delta T\frac{\bar{\rho}^{\gamma+1}}{\lambda a}$$

this proves the statement in Proposition 5.