

Demographic Evolutions and Unemployment: an analysis of French labour market with workers generations

Jean Chateau, Jean-Louis Guérin and Florence Legros

N° 2002-?
Octobre

Table of Contents

1. INTRODUCTION	11
2. ASSUMPTIONS ABOUT SOCIO-DEMOGRAPHIC PROSPECTS	12
2.1. The simple demographic structure of the model	12
2.2. Exogenous repartition between activity and inactivity	13
3. MODEL 1: ENDOGENOUS UNEMPLOYMENT WITH MATCHING	19
3.1. The matching process between unemployed and vacancies	20
3.2. The representative firm	21
3.3. Social security insurance systems	22
3.4. Calibration and dynamic simulation	23
4. MODEL 2: ENDOGENOUS UNEMPLOYMENT WITH MATCHING AND WAGE BARGAINING	28
4.1. Model improvements	28
4.2. Base case : calibration and simulation	29
4.3. Alternative reforms of the pensions system	33
5. CONCLUSION	37
A APPENDIX	38
A1. Exogenous quarterly probabilities of death and ageing	38
A2. Model 2. with Quadratic Vacancies Costs	39
A3. Sensitivity Analysis for Model 2.	40

Abstract

This paper describes a segmented matching labour market model with generations of workers. This model is calibrated on French data. The results refute the usual idea, according to which a decrease in labour supply leads to a decrease in unemployment. The combination of pure demographic effects with fiscal wedge effects due to the increase in subscription rates lead to a serious rise of the unemployment rate. Various alternative specifications are also studied such as an increase of work duration or a lowering in replacement rates.

Résumé

Ce document présente un modèle d'équilibre du marché du travail dans lequel les travailleurs appartenant à des classes d'âge différentes participent à des marchés du travail a priori segmentés. Ce modèle d'appariement est étalonné sur données françaises. Les résultats contredisent le présupposé habituel qui veut qu'à la raréfaction de la main d'œuvre doivent correspondre, d'ici quelques années, une baisse du chômage. La combinaison d'effets de composition purement démographiques, mais surtout les effets de type "coin fiscal" consécutifs à l'augmentation des taux de cotisation retraite entraînent en effet une forte hausse du chômage. Différents scénarios alternatifs sont présentés parmi lesquels l'allongement de la durée du travail ou la baisse des taux de remplacement.

J.E.L. classification number: J1, J41, J64, E24, H55.

Keywords: Ageing, Unemployment, Matching Model, Social Security.

Mots Clés : Vieillesse, Chômage, Modèle d'appariement, Sécurité Sociale.

Summary

People is often led to assume that the ageing process will imply short cuts in the active population and so, a decrease in the unemployment rates. On the one hand this assumption do not lie on an analysis of the various segments that composed the labour market (yongs, median aged workers and olds) and only suggest that the present low employment rates of the eldest workers would disappear with the labour growing scarcity. On the other hand, these analysis do not rely on the microeconomic foundations of the labour markets. An other problem is that institutional features are generally forgotten.

In this paper we have a mesoeconomic model in which the workers belong to different categories according to their age. Each category corresponds to a specific labour market, i.e. the labour markets are supposed to be segmented.

One of the reasons why we assume this segmentation is due to the reservation wage of each of the category; the alternative income that workers may receive differs according to their age: the yongs may attend the assistance income and the eldests a pre-retirement pension. To sum up, the bargaining power of each category differs from the others.

Our model is a simple matching model with segmented labour market ; we distinguish 5 agents groups: the young workers (less than 30), the median ones (30-55 years old), the 55 and older, the young inactives (less than 20) and olds (65 and more). The age pyramids evolve according to demographic projections made at CEPII while the activity rates are drawn from INSEE "Enquête emploi" (National statistics and economic studies institute labour force survey). There are 4 economic types of agents: firms, active workers, inactive people, the unemployment social insurance and the retirement pension scheme PAYG funded. The former is financially balanced by the way of contributions adjustments. The pre-retirement schemes are financed by the unemployment insurance according to the present rules.

On the one hand, the way the bargains are driven play an great role through the transition between activity status probabilities. On the other hand, the ageing impact on the social security institutions balances and, by the way, on the contribution rates have also a strong impact.

The demographic impact are the strongest when the transition probabilities remain constant (when the demographics do not change the bargaining between wage earners and firms) while the latter effect (the insurance balancing) is very strong when it is accompanied by a change in the bargaining play (model with endogenous wages). In this case, there is a strong increase in the unemployment rates for each workers' category. The huge increase in the contribution rates - due to the ageing process - implies, if the contributions are mainly paid by the wage earners i.e. if these contributions and the net

wages are substitutable, an increase in the fiscal wage that pushes up the unemployment. In the polar case, if the adjustments are made by the way of the replacement rates - the fiscal wage then disappears - we come back to the demographic impact preponderance.

J.E.L. classification number: J1, J41, J64, E24, H55.

Keywords: Ageing, Unemployment, Matching Model, Social Security.

Résumé Long

On est quelquefois naturellement conduit à penser que le vieillissement démographique s'accompagnant d'une baisse de la population active induira une réduction du taux de chômage mais, d'une part, ces analyses ne s'intéressent pas aux différents segments qui composent le marché du travail (jeunes, âges moyens, vieux travailleurs) et se contentent de suggérer que les difficultés actuelles des travailleurs âgés pourraient disparaître de facto avec la raréfaction du facteur travail ; d'autre part, ces analyses ne s'intéressent que peu aux fondements microéconomiques des évolutions des marchés du travail. Par ailleurs, les études empiriques font souvent l'impasse sur les caractéristiques insitutionnelles.

Nous avons fait le choix d'une modélisation mésoéconomique. Le modèle est un modèle d'équilibre dans lequel les travailleurs appartenant à une classe d'âge différente participent à des marchés du travail a priori segmentés.

L'une des raisons pour lesquelles nous avons choisi de segmenter la pyramide des âges actives en plusieurs marchés du travail tient notamment au salaire de réservation : le revenu alternatif que peuvent attendre des agents différent selon leur âge ; les jeunes peuvent avoir accès au RMI et les plus âgés à une pension dite de " préretraite ", par exemple. Autrement dit, la capacité de négociation des différentes classes d'âge est différente.

Notre modèle se situe dans un cadre simple d'appariement avec marché du travail segmenté ; nous distinguons 5 catégories d'agents : les travailleurs jeunes (moins de 30ans), les 30-55 ans, les plus de 55 ans, les inactifs jeunes (moins de 20 ans) et vieux (plus de 65 ans). Les projections démographiques utilisées pour faire évoluer la pyramide des âges sont celles qui résultent des travaux effectués au CEPII. Les taux d'activité résultent des probabilités de transition issues de l'enquête emploi. Les " acteurs économiques " sont de 4 types : les entreprises, les actifs-inactifs, catégories auxquelles il faut ajouter une caisse d'assurance-chômage (y compris pré-retraites) ainsi qu'une caisse d'assurance vieillesse. L'équilibre de cette dernière se fait mécaniquement par l'ajustement des taux de cotisation.

On a donc des effets qui dépendent, d'une part, de la manière dont s'effectuent les négociations (ce qui influe sur les probabilités de transition entre états sur le marché du travail), d'autre part, de l'incidence du vieillissement et des passages par la préretraite et le chômage sur l'équilibre des caisses d'assurance (respectivement, retraite et chômage-préretraite). Les effets démographiques sont prépondérants dans le cas où les probabilités de transition entre états restent constantes (i.e dans le cas où la démographie ne modifie pas les rapports de force entre salariés et entreprises) alors que les effets liés au rééquilibrage des caisses d'assurance sont prépondérants dans le cas où il y a modification des rapports de force (modèle avec endogénéisation des salaires) :

dans ce cas, il y a augmentation du chômage pour l'ensemble des catégories de travailleurs. L'augmentation massive des cotisations - suite au processus de renflement aux âges élevés de la pyramide des âges - a pour effet - si les cotisations sont à la charge des actifs, i.e. si il y a substituabilité entre cotisations sociales et salaires nets - une augmentation du coin fiscal (la différence entre le salaire net et le coût du travail) qui se solde par une augmentation du taux de chômage. Dans le cas polaire, si les ajustements se font par un ajustement des taux de remplacement - il n'y a dans ce cas plus d'effet de coin fiscal - on retrouve les effets purement démographiques.

Classification JEL : J1, J41, J64, E24, H55.

Mots Clés : Vieillissement, Chômage, Modèle d'appariement, Sécurité Sociale.

Demographic Evolutions and Unemployment : an analysis of French labour market with worker generations¹

Jean Chateau, Jean-Louis Guérin and Florence Legros²

1. INTRODUCTION

The forthcoming demographic evolutions in industrialised countries raise many questions. Will the slow down of the working age population lead unemployment rate to be lowered? Will workers ageing give way to an evolution of the wage setting process by skill and seniority? Between volume and composition effects, what will have the strongest impact? Finally, will the pensions system be able to withstand these changes or shall we see a conflict between generations? Those questions have been extensively studied. Pensions were the object of the first Report by the *Conseil d'Orientation des Retraites* (2001) or in Gruber and Wise (1999). Consequences of this demographic turn and some potential solutions to deal with are also looked at in the Quintreau (2001) report and in the forthcoming report from the "prospects of jobs and skills" group of the *Commissariat Général du Plan*. Questions over the general macroeconomic consequences of demographic evolutions have been studied by Domingues Dos Santos (2001), Blanchet (2001) and a report of the *Conseil d'Analyse Economique* (Aglietta, Blanchet and Hérin 2002).

A previous work by Cepii focused on the labour markets issues (Cadiou, Genet and Guérin 2000). This research was conducted in three steps by first looking at the impacts of demographic changes on global demand and on savings-consumption sharing, so in a rather Keynesian approach. This first step underlined the numerous uncertainties, as age impacts on the consumption structure are generally lower than thought (specifically when compared to generation effects), and as the impacts of demographic evolutions on activity and savings strongly depend on the institutional changes that will be made to enable pensions systems to adapt. The second step followed this track in a classical unemployment framework, so focusing on wage setting mechanisms. While some specific impacts may appear transitionally, the models rather credit the idea of a relative independence in the long run between the unemployment rate and demographic variables. Finally, the last part of this study looked closely at the differences in characteristics and behaviours by age, so focusing on composition effects. Those were shown

¹The authors acknowledges financial support from CEPII as well as the Institut de la Caisse des dépôts pour la recherche économique. The authors want to express their thanks to Fabien Postel-Vinay for his help in the elaboration of the model, Rakhsat Sleiman for the construction of demographic prospects and Béatrice Sédillot for enabling us access to data from Insee *Enquêtes Emploi*. Authors also benefited from comments of participants in Enepri and CDC workshops in Nantes (Sep. 2001) and Bordeaux (Sep. 2002). Correspondence : chateau@cepii.fr.

²CEPII (9, rue Georges Pitard, 75740 Paris Cedex 15, France).

to be potentially important, but the study concluded at the endogenous nature of some of them (seniority wage relationships for example).

The current paper pursues this line of thought and aims to describe these effects in a matching labour market model with generations of workers calibrated on French data. As a matter of fact, this paper doesn't consider ageing and labor market participation from the household point of view as in Stock and Wise (1990). On the contrary, we insist on the relation between firms and households behaviour about employment and wages setting, along the line of Layard, Nickell and Jackman (1991), and its plausible evolution with ageing. This paper presents an equilibrium model with age-segmented labour markets. However, demographic connections between those sub-markets lead to some transmissions of impacts. This segmentation hypothesis may be discussed, both at supply and demand levels. But, a relative heterogeneity between workers by age is a reality (productivity, bargaining power, unionization rates, experience, mobility, learning capacity, etc.). At the same time, many jobs offers are age focused. We are aware that those disparities are less important than those between skilled and unskilled, and aim to cross those two approaches in the future.

The paper is structured as follows: the next section presents the demographic and active population prospects for France used in the paper; the third one describes the matching model, the calibrations procedures and the expected impacts; the fourth section introduces a wage bargaining process and gives the results for alternative macroeconomic or institutional closure rules; the fifth section concludes. Technical details can be found in the appendix.

2. ASSUMPTIONS ABOUT SOCIO-DEMOGRAPHIC PROSPECTS

The time unit considered in the paper is the quarter of a year. Unless otherwise indicated, transition probabilities are quarterly probabilities.

2.1. The simple demographic structure of the model

Five age groups are considered: the children (denoted by e), the people over the mandatory retirement age (r) and the working age population; in turn, this last class is composed of young adults (j), middle aged adults (m) and seniors (v). Transition between those various groups is supposed to happen at the beginning of each period, before any evolution in activity status. By convention, the evolution of the population of an age group at the beginning of period t is given by the following formula (with " a " the next younger age group) for $a = j, m, v, r$:

$$P_t^a = P_{t-1}^a \mu_{t-1}^a \cdot (1 - \tilde{\lambda}_{t-1}^a) + \tilde{\lambda}_{t-1}^{a-i} \mu_{t-1}^{a-i} P_{t-1}^{a-i}$$

where $1 - \mu_{t-1}^a$ is the proportion of class " a " individuals who died during the period $t-1$ (i.e. between $t-1$ and t), and $\tilde{\lambda}_{t-1}^a$ is the proportion of individuals who survived the period $t-1$ and who became older and changed age group at t . The death corrected

probability of changing age group will be denoted $\lambda_{t-1}^a = \mu_{t-1}^a \tilde{\lambda}_{t-1}^a$. So, the laws of motion for each age-group can be written as (exogenous variables are over lined):

$$P_t^j = P_{t-1}^j(\bar{\mu}_{t-1}^j - \bar{\lambda}_{t-1}^j) + \bar{\lambda}_{t-1}^e \bar{P}_{t-1}^e \quad (1)$$

$$P_t^m = P_{t-1}^m(\bar{\mu}_{t-1}^m - \bar{\lambda}_{t-1}^m) + \bar{\lambda}_{t-1}^j P_{t-1}^j \quad (2)$$

$$P_t^v = P_{t-1}^v(\bar{\mu}_{t-1}^v - \bar{\lambda}_{t-1}^v) + \bar{\lambda}_{t-1}^m P_{t-1}^m \quad (3)$$

$$P_t^r = P_{t-1}^r \bar{\mu}_{t-1}^r + \bar{\lambda}_{t-1}^v P_{t-1}^v \quad (4)$$

The simulations start from the first quarter of year 2001; at that stage $t=0$, populations (1)–(4) are known. Future populations can then be calculated, using the transition probabilities $\{\bar{\mu}_{t-1}^j, \bar{\lambda}_{t-1}^j\}_{t>0}$ (with $\bar{\mu}_0^j$ being the transition probability between December 2000 and March 2001) and the number of new entrants $\{\bar{P}_{t-1}^e\}_{t>0}$. Those figures are derived from the total population prospects for Metropolitan France obtained with the death prospects method developed at INED by Meslé and Vallin (2001). The following hypotheses are done: no migrants flows, values of instant fecundity index and median maternity age maintained at 1.76 children by woman and 30.8 years. The time scope of these prospects is the first quarter of year 2049. After this date, the exogenous parameters are considered to be invariant (Figure 1). Those figures show the spectacular increase of people aged over 65. The temporary increase in seniors workers numbers due to the baby boom can also be seen. The corresponding exogenous probabilities can be found in the appendix.

2.2. Exogenous repartition between activity and inactivity

For the moment, labour market participation behaviours are exogenous. The segmentation of the working age population between active and inactive can be modelled in various ways³. All must however be consistent with the following definition: $P_t^a \equiv L_t^a + I_t^a$ ($\forall t$ and $a = j, m, v$), where L_t^a is the active population of age group a and I_t^a the inactive. All individuals over 65 will be supposed to be retired. The activity rate of age group a at t will be denoted by I_t^a . The activity decision of an individual is supposed to take place after age evolution. Eqs (5)–(6) give the active populations of young and middle age groups.

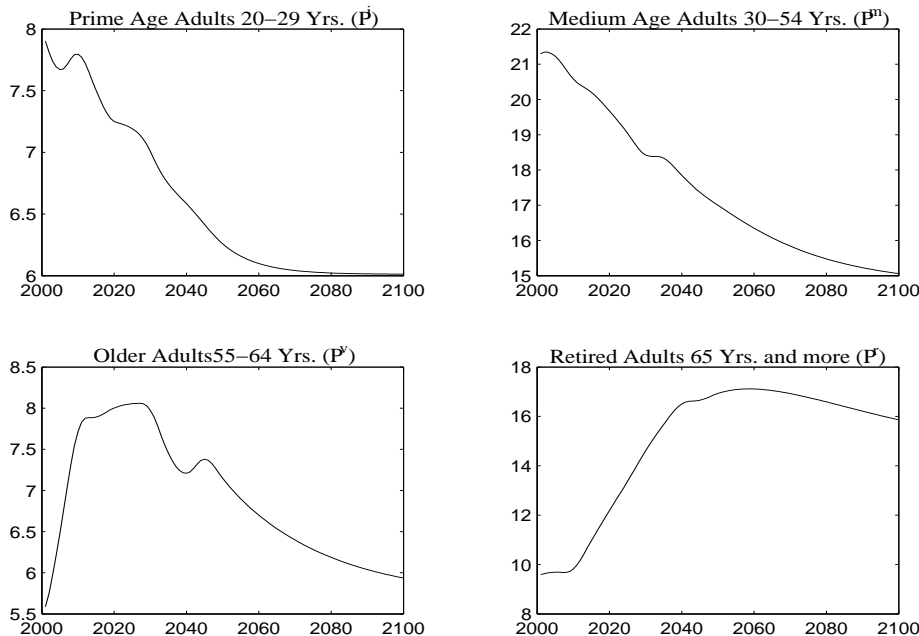
$$L_t^j = (\bar{\mu}_{t-1}^j - \bar{\lambda}_{t-1}^j)[(1 - \bar{\gamma}_{t-1}^j)L_{t-1}^j + \bar{\xi}_{t-1}^j(P_{t-1}^j - L_{t-1}^j)] \\ + [\bar{t}_{t-1}^e(1 - \bar{\gamma}_{t-1}^j) + (1 - \bar{t}_{t-1}^e)\bar{\xi}_{t-1}^j]\bar{\lambda}_{t-1}^e \bar{P}_{t-1}^e \quad (5)$$

$$L_t^m = (\bar{\mu}_{t-1}^m - \bar{\lambda}_{t-1}^m)[(1 - \bar{\gamma}_{t-1}^m)L_{t-1}^m + \bar{\xi}_{t-1}^m(P_{t-1}^m - L_{t-1}^m)] \\ + \bar{\lambda}_{t-1}^j(1 - \bar{\gamma}_{t-1}^m)L_{t-1}^j + \bar{\lambda}_{t-1}^j \bar{\xi}_{t-1}^m(P_{t-1}^j - L_{t-1}^j) \quad (6)$$

Where γ_{t-1}^a and ξ_{t-1}^a being respectively the probabilities for a class a active (inactive)

³The details of those choices and their implications can be found in a technical paper.

Figure 1: Population Prospects by Age Groups (millions)



having survived at t of becoming inactive (resp. active) during this period. So new entrants are considered to have the same transition probabilities (but not the same activity rates) as their new age class. The specific case of the children becoming of working age needs to be addressed as they were all inactive previously. The parameter t_t^e is the exogenous activity rate of those new entrants in the youths group⁴. Things are more complicated for seniors, due to the possibility of early or pre-retirement.

Retired and pre-retired people

One of the difficulties raised by the age segmentation process comes from the fact that between 50 and 65, activity cessation is a complex process that is not perfectly in line with age. From an institutional point of view, the entry into retirement is not always time uniform, nor direct, due to the presence of pre-retirement schemes. During the 90's, only two people out of five clearing their pension under the "general regime" (CNAV) were employed active people. The others were either unemployed, or inactive or pre-retired. This last status could be thought to entail no possibility of ever working again, and so of a similar treatment as an age group evolution.

However, things are more complicated as a large proportion of senior unemployed people are not required to search for a job (called "DRE") and belong in fact to this category. The number of people benefiting from an official pre-retirement scheme

⁴The details of those choices and their implications can be found in a technical paper.

("State" schemes, *ARPE* and *CATS*) is 190300. Added to the 348824 *DRE*, the effective size of this category at the 12/31/2000 is 539124. As almost all of them are between 55 and 64 years old, and as very few people do interrupt their careers before 55, the senior age group will be considered to be between 55 and 64 years old, and not over 50 as is usually done⁵. The population of this age group can so be segmented between:

1. Active workers: (N^v),
2. Active Unemployed : U^v ,
3. Inactive people liable to becoming active : I^v ,
4. Former active people having left activity early (whether *DRE* or pre-retired), not liable to entering the labour market again, and whose replacement revenue is paid by the unemployment benefits agency (Unedic): PR^v ,
5. Retired people having liquidated their pension before the age of 65 : R^v .

Between two dates, a class "v" individual can leave the labour market definitively, either by entering pre-retirement, or by entering retirement, or by reaching the maximum legal retirement mandatory age (65 years). Once pre-retired, an individual can only evolve towards retirement. An accurate description of the passage from pre-retirement to retirement for class v individuals would require to clearly distinguish between early retirement and job quitting, and to identify the underlying reasons. At this stage, exogenous transition probabilities are considered: ζ_{t-1}^{pr} is the probability of entering pre-retirement, ζ_{t-1}^r of entering retirement while active, and ς_{t-1} of shifting from pre-retirement to retirement (eliminating inactive I^v from $P_t^v \equiv L_t^v + I_t^v + R_t^v + PR_t^v$).

$$\begin{aligned} L_t^v &= (\mu_{t-1}^v - \lambda_{t-1}^v)(1 - \gamma_{t-1}^v - \zeta_{t-1}^{pr} - \zeta_{t-1}^r)L_{t-1}^v \\ &+ \lambda_{t-1}^m[L_{t-1}^m(1 - \gamma_{t-1}^v - \xi_{t-1}^v) + \xi_{t-1}^v P_{t-1}^m] \\ &+ (\mu_{t-1}^v - \lambda_{t-1}^v)\xi_{t-1}^v(P_{t-1}^v - L_{t-1}^v - R_{t-1}^v - PR_{t-1}^v) \end{aligned} \quad (7)$$

$$PR_t^v = (\mu_{t-1}^v - \lambda_{t-1}^v)[(1 - \varsigma_{t-1})PR_{t-1}^v + \zeta_{t-1}^{pr}L_{t-1}^v] \quad (8)$$

$$R_t^v = (\mu_{t-1}^v - \lambda_{t-1}^v)[R_{t-1}^v + \varsigma_{t-1}PR_{t-1}^v + \zeta_{t-1}^rL_{t-1}^v] \quad (9)$$

As indicated by these equations, a middle aged worker entering the senior group can not instantaneously quit his job definitively. Using previous data and hypothesis, and the general demographic evolution laws being given, it is possible to determine the future evolutions of active population by age group. The calibration of the model is based on data from the *Enquête Emploi* from INSEE. These data enable the calculation of the transition probabilities between activity and inactivity (Table 1). Transition probabilities between pre-retirement and retirement are derived from the average allowance duration for pre-retired given by the unemployment insurance Assedic ($\bar{\varsigma}_0 = 0.085$). The complexity of the French Pay-as-you-go (PAYG) system makes it difficult to determine precisely the number of pensioned people between 55 and 64. An evaluation by the Drees gives a number of retired of 12.1 millions people (reversion pensions included) in 2000 (Mesnard 2001). The annual report of the Cnav (2001) report indicates

⁵In the same way, the youths age group covers the 20 to 29 years old, and not the usual under 35.

Table 1: Activity Calibration

Quarterly Transition probabilities										
ξ^j	=	0.067094707	ξ^m	=	0.04251595	ξ^v	=	0.007417072		
γ^j	=	0.016239698	γ^m	=	0.008621448	γ^v	=	0		
Activity rates at t=0				Retirement and Pre-retirement						
Age	e	j	m	v	\bar{R}_0^v	=	2250100	$\bar{P}R_0^v$	=	539124
Group	32.8	68.7	85.9	37.3	\bar{R}_0^r	=	9597012	ζ_0^{pr}	=	0.085

that 18.59% of the beneficiaries of the general regime were between 55 and 65 years old at the end of 2000 (or a proportion of 32.7% of this age group), the large majority of the others being over 65 years old. Applying those proportions to the total number of retired gives $\bar{R}_0^v = 2250100$ and $\bar{R}_0^r = 9849900$. That last figure is slightly higher to the initial total population of age group "r" (9 597 012). The most likely explanation is that the share of the 55-65 years old is underestimated when taking into account all pensions systems⁶. As a simplification measure, the figure for 55-65 is kept on and it is supposed that all people over 65 are retired and do receive a pension ($R_t^r = P_t^r$). On that basis, and given the average activity rates of *Enquête Emploi* (Table 1), equations (5)–(9) give us the initial sizes of the active population consistent with those parameters, as well as the transition probabilities towards job quitting (Table 2).

Table 2: Calculated Activity Variables

L_0^j	=	5 425 700	ζ_0^{pr}	=	0.032
L_0^m	=	18 301 000	ζ_0^r	=	0.021
L_0^v	=	2 068 500			

The authors of the micro-simulation model Destinie (1999) at INSEE, find a yearly probability of job quitting after 55 of 15%. Our calibration gives a result of 13.5%. Part of the small difference between the two estimates stems from the lowering of pre-retirement schemes since 1999, date of the Destinie evaluations. The yearly probability of shifting directly from activity to retirement before reaching the maximum mandatory retirement age is 8.5%.

Results of those simulations are given in figures 2–3. The observed decrease in the activity rates between 30 and 64 is not balanced by the small increase of this rate for the young. Those evolutions are not monotonous as there are some relatively large cohorts in the initial sample. During the next 50 years, total active population should

⁶This error comes first from the acknowledged approximate nature of the Drees evaluation. Additionally, there is no reason for the different pensions schemes to have the same age structure as the general regime. For example, public servants often enter retirement earlier than beneficiaries of the general regime, which in turn quit job earlier than unsalaried people (Charpin 1999). What's more, the presence of beneficiaries under 55 (ex military) and the existence of reversions pensions can also explain part of that bias.

decrease from 25 to 20 millions (there are no migratory flows considered). During the same period, the number of retired people will almost double, rising from 12 to 20 millions. Such an evolution of the active population differs from the prospects done by Brondel, Guillemot, Lincot and Marioni (1996) who found a decrease in the youths and seniors activity rate and a rise for middle aged people. That is because their activity prospects are done by extrapolating activity behaviours over the period 1968-95 (increase of school duration and in early job quitting) before applying them to demographic forecasts. Whereas in this paper, activity behaviours as described by transition probabilities are fixed at their values of 2000-01, so in a rather dynamic period relatively to the observation period used by Insee.

Figure 2: Active and Retired Population by Age-Group : Individuals (in millions)

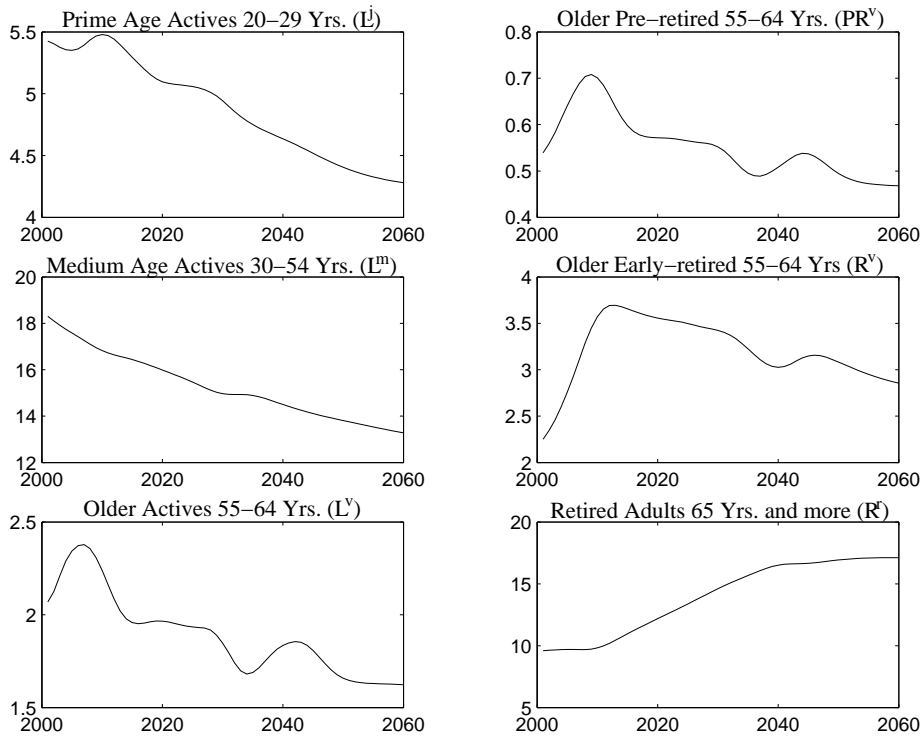
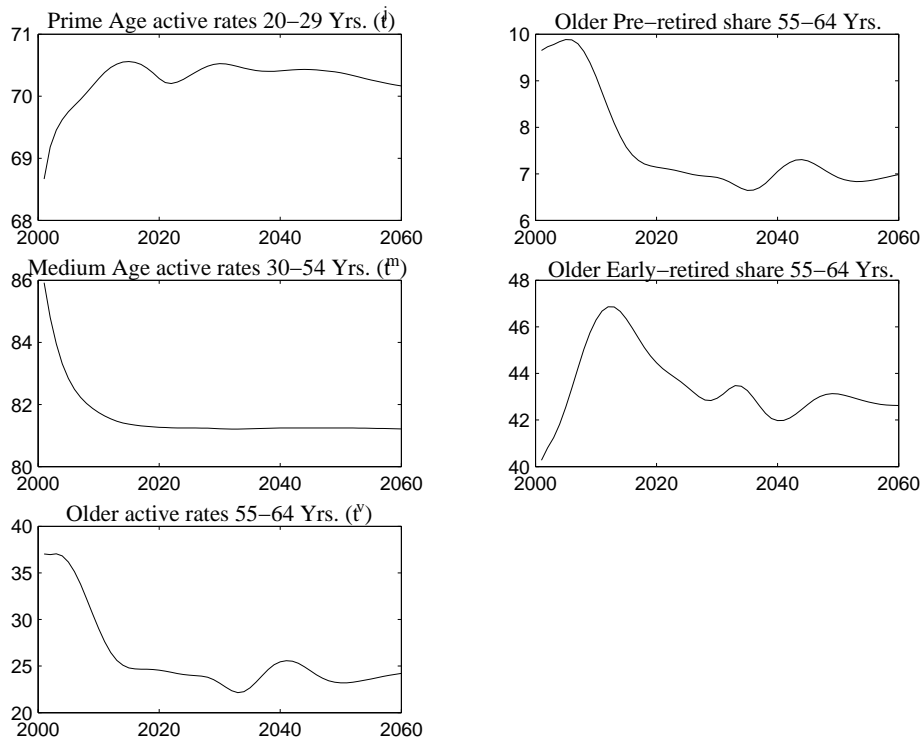


Figure 3: Active and Retired Population by Age-Group : Rates (percentage of total Age-Group population)



3. MODEL 1: ENDOGENOUS UNEMPLOYMENT WITH MATCHING

At the beginning of each period, an individual can either be unemployed ($\in U^a$), or at work ($\in N^a$). In this last case, we are in line with Pissarides (1990) by supposing that, with the probability $\bar{\delta}_{t-1}^a$ his wage-contract is not extended consecutive to an exogenous job destruction. In the first case, the probability of finding a job is p_t^a . By definition of the active population, we do have: $L_t^a \equiv U_t^a + N_t^a$ ($\forall t$ and $a = j, m, v$). Employment is determined at the beginning of the period t (but after demographic and activity evolutions) for the entire period. Job finding for an unemployed occurs at that time, whereas job destruction happens at the end of the period. Applying such a sequential dynamic process to former active people who shift towards the older age group by ageing does not rise any problems as those people, whose former work status is known, now use the transition probabilities specific to their new age category ($\lambda_{t-1}^{a-1} \xi_{t-1}^a I_{t-1}^{a-1}$). Things are a little more complex for former inactive people as we need to make an hypothesis on their dispatch between work and unemployment. Following the model's line, they ought to be considered as unemployed, at least for a time; however, so as to simplify, those people will be supposed to have the same probabilities of finding a job as unemployed people. The same reasoning is used for former children, all of whom were inactive, and who become adults. As $L_t^a \equiv U_t^a + N_t^a$, workers can be left out (like inactive):

$$\begin{aligned} U_t^j &= (\bar{\mu}_{t-1}^j - \bar{\lambda}_{t-1}^j)(1 - \bar{\gamma}_{t-1}^j)[\bar{\delta}_t^j L_{t-1}^j + (1 - p_t^j - \bar{\delta}_t^j)U_{t-1}^j] \\ &+ (\bar{\mu}_{t-1}^j - \bar{\lambda}_{t-1}^j)\bar{\xi}_{t-1}^j(1 - p_t^j)(P_{t-1}^j - L_{t-1}^j) \\ &+ (1 - p_t^j)[\bar{t}_{t-1}^e(1 - \bar{\gamma}_{t-1}^j) + (1 - \bar{t}_{t-1}^e)\bar{\xi}_{t-1}^j]\bar{\lambda}_{t-1}^e \bar{P}_{t-1}^e \end{aligned} \quad (10)$$

$$\begin{aligned} U_t^m &= (\bar{\mu}_{t-1}^m - \bar{\lambda}_{t-1}^m)(1 - \bar{\gamma}_{t-1}^m)[\bar{\delta}_t^m L_{t-1}^m + (1 - p_t^m - \bar{\delta}_t^m)U_{t-1}^m] \\ &+ (\bar{\mu}_{t-1}^m - \bar{\lambda}_{t-1}^m)\bar{\xi}_{t-1}^m(1 - p_t^m)(P_{t-1}^m - L_{t-1}^m) \\ &+ \bar{\lambda}_{t-1}^j(1 - \bar{\gamma}_{t-1}^m)[(\bar{\delta}_t^m L_{t-1}^j + (1 - p_t^m - \bar{\delta}_t^m)U_{t-1}^j)] \\ &+ \bar{\lambda}_{t-1}^j \bar{\xi}_{t-1}^m(1 - p_t^m)(P_{t-1}^j - L_{t-1}^j) \end{aligned} \quad (11)$$

$$\begin{aligned} U_t^v &= (\bar{\mu}_{t-1}^v - \bar{\lambda}_{t-1}^v)(1 - \bar{\gamma}_{t-1}^v - \zeta_{t-1}^{pr} - \zeta_{t-1}^r)(\bar{\delta}_t^v L_{t-1}^v + (1 - p_t^v - \bar{\delta}_t^v)U_{t-1}^v) \\ &+ (\bar{\mu}_{t-1}^v - \bar{\lambda}_{t-1}^v)\bar{\xi}_{t-1}^v(1 - p_t^v)(P_{t-1}^v - L_{t-1}^v - R_{t-1}^v - PR_{t-1}^v) \\ &+ \bar{\lambda}_{t-1}^m(1 - \bar{\gamma}_{t-1}^v)[\bar{\delta}_t^v L_{t-1}^m + (1 - p_t^v - \bar{\delta}_t^v)U_{t-1}^m] \\ &+ \bar{\lambda}_{t-1}^m \bar{\xi}_{t-1}^v(1 - p_t^v)(P_{t-1}^m - L_{t-1}^m) \end{aligned} \quad (12)$$

Unemployed people in the "a" age group at t include the former unemployed who survived and didn't change age group nor became inactive nor found a job, plus the unemployed of the next younger age group who became older but didn't find a job at the beginning of the period, plus the newly unemployed: former workers of this age group who lost their job at the end of the previous period, plus the former inactive people (whether of this or of the next younger age group) who didn't succeed in finding a job. When transition probabilities \bar{p}^a and $\bar{\delta}^a$ are known, eqs (10)–(12) give the initial unemployment level for each age group. On the reverse, those equations can be used to

reproduce the effective unemployment rates (ILO definition) given by *Enquête Emploi* data at $t=0$ (Table 3) letting some transition probabilities adjust. As the probabilities of job finding \bar{p}^a are well documented (from average lengths of unemployment are given by *Unedic*), the job destruction probabilities $\bar{\delta}^a$ are here the variables calculated so as to reproduce the initial unemployment rates.

Table 3: Unemployment Calibration			
Quarterly Transition Probabilities			Unemployment rates in $t=0$
\bar{p}^j	= 0.5	\bar{p}^m	= 0.286
\bar{p}^v	= 0.167		
			Age
			Group
			j
			m
			v
			16.6
			8.05
			4.48
Calculated Quarterly Destruction Rates			
$\bar{\delta}^j$	= 0.0816	$\bar{\delta}^m$	= 0.0198
$\bar{\delta}^v$	= 0.0035		

The corresponding rate of job destruction found for senior workers is very weak. This comes in part from the non consideration of *DRE*-unemployment and pre-retirement schemes. Let's note that the weighted average destruction probability in the economy is 13.2%, which is very close to the 13.9% given by Cahuc and Zylberberg (1996) (p.426).

3.1. The matching process between unemployed and vacancies

After $t=0$, job finding probabilities become endogenous. Standard modelling uses matching functions giving the number of M^a considering the number of unemployed job seekers \tilde{U}^a and the number of vacancies V^a offered by the firms. Filling a vacancy is a costly process for the firm both in time and in resources, due in part to the existence of information asymmetries (Pissarides 1990). At a macro level, the search activity can be thought as the result of a process taking into account labour market's imperfections described by the following matching functions:

$$M_t^j = \bar{A}_M^j (V_t^j)^{\phi^j} \cdot (\tilde{U}_t^j)^{1-\phi^j} \quad (13)$$

where $\tilde{U}_t^j = (\bar{\mu}_{t-1}^j - \bar{\lambda}_{t-1}^j)[(1 - \bar{\gamma}_{t-1}^j)U_{t-1}^j + \bar{\xi}_{t-1}^j(P_{t-1}^j - L_{t-1}^j)] + [\bar{t}_{t-1}^e(1 - \bar{\gamma}_{t-1}^j) + (1 - \bar{t}_{t-1}^e)\bar{\xi}_{t-1}^j]\bar{\lambda}_{t-1}^e \bar{P}_{t-1}^e$,

$$M_t^m = \bar{A}_M^m (V_t^m)^{\phi^m} \cdot (\tilde{U}_t^m)^{1-\phi^m} \quad (14)$$

where $\tilde{U}_t^m = (\bar{\mu}_{t-1}^m - \bar{\lambda}_{t-1}^m)[(1 - \bar{\gamma}_{t-1}^m)U_{t-1}^m + \bar{\xi}_{t-1}^m(P_{t-1}^m - L_{t-1}^m)] + \bar{\lambda}_{t-1}^j[(1 - \bar{\gamma}_{t-1}^m)U_{t-1}^j + \bar{\xi}_{t-1}^m(P_{t-1}^j - L_{t-1}^j)]$,

$$M_t^v = \bar{A}_M^v (V_t^v)^{\phi^v} \cdot (\tilde{U}_t^v)^{1-\phi^v} \quad (15)$$

where $\tilde{U}_t^v = (\bar{\mu}_{t-1}^v - \bar{\lambda}_{t-1}^v)[(1 - \bar{\gamma}_{t-1}^v - \zeta_{t-1}^{pr} - \zeta_{t-1}^r)U_{t-1}^v + \bar{\xi}_{t-1}^v(P_{t-1}^v - L_{t-1}^v - R_{t-1}^v - PR_{t-1}^v)] + \bar{\lambda}_{t-1}^m[(1 - \bar{\gamma}_{t-1}^v)U_{t-1}^m + \bar{\xi}_{t-1}^v(P_{t-1}^m - L_{t-1}^m)]$.

The hypothesis of M being globally linearly homogenous is now commonly used in the

literature (Blanchard and Diamond 1990)). Following applied works, we suppose, as in Mortensen and Pissarides (1999), that $\phi^a = 0.5$. In the rest of the paper, $\theta^a = V^a/\tilde{U}^a$ will be the labour market tightness indicator (in the sense that when wage will be endogenous there will be rising functions of θ^a).

The average probabilities of finding a job p^a and of filling a vacancy q^a can then be defined as:

$$p_t^a = \frac{M_t^a}{\tilde{U}_t^a} \quad q_t^a = \frac{M_t^a}{V_t^a}$$

p^a rises with θ^a , whereas q^a decreases with θ^a . The constant returns to scale of the matching functions enable to link directly those probabilities of transition with tightness on a segment of the labour market: $p^a(V^a/\tilde{U}^a) = (V^a/\tilde{U}^a)q^a(V^a/\tilde{U}^a)$. As those probabilities are taken as given by competitive agents, the interaction between the stocks of unemployed labour and the flows out of unemployment is not taken into account by the various actors. The existence of those congestion externalities is an important characteristic of the model.

The equations describing the evolution of workers stocks can then be derived by incorporating those probabilities in the equations of the previous section (Pissarides 1990) :

$$N_t^j = (\bar{\mu}_{t-1}^j - \bar{\lambda}_{t-1}^j)(1 - \bar{\gamma}_{t-1}^j)(1 - \bar{\delta}_t^j)N_{t-1}^j + q_t^j V_t^j \quad (16)$$

$$N_t^m = (1 - \bar{\gamma}_{t-1}^m)(1 - \bar{\delta}_t^m)[(\bar{\mu}_{t-1}^m - \bar{\lambda}_{t-1}^m)N_{t-1}^m + \bar{\lambda}_{t-1}^j N_{t-1}^j] + q_t^m V_t^m \quad (17)$$

$$N_t^v = (1 - \bar{\delta}_t^v)(\bar{\mu}_{t-1}^v - \bar{\lambda}_{t-1}^v)(1 - \bar{\gamma}_{t-1}^v - \bar{\zeta}_{t-1}^{pr} - \bar{\zeta}_{t-1}^r)N_{t-1}^v + (1 - \bar{\delta}_t^v)\bar{\lambda}_{t-1}^m(1 - \bar{\gamma}_{t-1}^v)N_{t-1}^m + q_t^v V_t^v \quad (18)$$

3.2. The representative firm

So as to simplify, it is supposed that there exists a representative firm whose aim is to determine unilaterally (*right to manage* hypothesis in the words of Nickell and Andrews (1983)), the numbers of hires and of vacancies offered for each age group; the firm aims to maximise the expected value of her inter-temporal profit under constraints (16)–(18), taking as given the various transition probabilities and costs factors:

$$\max_{\{N_t^a, V_t^a\}_{t>0} \forall a} \Pi(1) = E_1 \sum_{t>0} \bar{\beta}^{t-1} (\pi_t - C_t^j(V_t^j) - C_t^m(V_t^m) - C_t^v(V_t^v) - CF^j \cdot M_t^j - CF^m \cdot M_t^m - CF^v \cdot M_t^v),$$

with $\pi_t = F(A_{t-1}^j N_{t-1}^j, A_{t-1}^m N_{t-1}^m, A_{t-1}^v N_{t-1}^v) - w_t^j N_{t-1}^j - w_t^m N_{t-1}^m - w_t^v N_{t-1}^v$, the current profits flow at date t stemming from the labour of the workers employed at the beginning of the period. $F(A_{t-1}^j N_{t-1}^j, A_{t-1}^m N_{t-1}^m, A_{t-1}^v N_{t-1}^v)$ is the labour using production technology of the only good. A_{t-1}^a is the specific average productivity of a worker of age group a at the beginning of the period t. $C_t^a(\cdot)$ is the convex exogenous hiring cost function (it can also be thought of as the cost of offering a vacancy while

looking for an adequate match). An additional cost $CF_t^a(\cdot)$ is paid in case of a successful hire (can be thought of as a training cost). w_t^a is the real gross wage paid at the end of the period. β is the discount factor⁷.

Under rational expectations hypothesis, the problem's solution can easily be derived from the adequate Lagrangian (with Φ_t^a the discounted Lagrange multipliers associated to constraints (16)–(18)) and is given by the following equations (to which must be added the transversality conditions) :

$$0 = \frac{\partial C_t^a(V_t^a)}{\partial V_t^a} - (\Phi_t^a - CF_t^a) \cdot q_t^a \quad \text{pour } a = j, v, m \quad (19)$$

$$\begin{aligned} \frac{\Phi_t^j}{\beta} &= A_t^j F_1' - w_{t+1}^j + \Phi_{t+1}^j (\bar{\mu}_t^j - \bar{\lambda}_t^j) (1 - \bar{\gamma}_t^j) (1 - \bar{\delta}_{t+1}^j) \\ &+ \Phi_{t+1}^m \bar{\lambda}_t^j (1 - \bar{\gamma}_t^m) (1 - \bar{\delta}_{t+1}^m) \end{aligned} \quad (20)$$

$$\begin{aligned} \frac{\Phi_t^m}{\beta} &= A_t^m F_2' - w_{t+1}^m + \Phi_{t+1}^m (\bar{\mu}_t^m - \bar{\lambda}_t^m) (1 - \bar{\gamma}_t^m) (1 - \bar{\delta}_{t+1}^m) \\ &+ \Phi_{t+1}^v \bar{\lambda}_t^m (1 - \bar{\gamma}_t^v) (1 - \bar{\delta}_{t+1}^v) \end{aligned} \quad (21)$$

$$\frac{\Phi_t^v}{\beta} = A_t^v F_3' - w_{t+1}^v + \Phi_{t+1}^v (\bar{\mu}_t^v - \bar{\lambda}_t^v) (1 - \bar{\gamma}_t^v - \bar{\zeta}_t^{Pr} - \bar{\zeta}_t^r) (1 - \bar{\delta}_{t+1}^v) \quad (22)$$

These equations look similar to the ones found by Mortensen and Pissarides (1999) in the case of a single labour market. The Lagrange multipliers can be seen as the expected value of the profit due to filling an additional job at date t . For the time being, the production function is supposed to combine perfectly substitutable kinds of effective labour and is linearly homogenous ($F_1' = F_2' = F_3' = 1$ at each period). The condition (19) is equivalent to the firms *free entry* condition on the labour market when firm can have only one job, filled or unfilled. Under this condition, the firm creates jobs as long as the expected marginal value of employment is equal to the cost of vacancy, at the equilibrium.

3.3. Social security insurance systems

The unemployment (and pre-retirement) benefits system (*Unedic*)

Unemployment benefits are funded by workers contributions (employers contributions are considered in a later part of the paper). The budget constraint is assumed to be time to time balanced. For this equilibrium (23 between the benefits and the subscriptions to be reached, it is required one of the variable adjusts over time: the subscription rate τ_t^u is the adjustment variable.

⁷This discount factor reflects the degree of impatience of the firm's owners, or the expected return rate of delaying consumption. In the model, the households instant utility function is linear, which means that agents are risk-neutral, leading to a constant expected return rate to consumption.

$$\tau_t^u \cdot (w_t^j N_{t-1}^j + w_t^m N_{t-1}^m + w_t^v N_{t-1}^v) = (1 - \tau_t^u) (b_t^j U_{t-1}^j + b_t^v U_{t-1}^v + b_t^m U_{t-1}^m + b_t^{pr} PR_{t-1}^v), \quad (23)$$

b_t^a is the (gross) average unemployment benefit. So as to simplify, it is supposed that benefits amounts, which do depend in fact on the past wages profile, are a function of the current average wage of the age group considered. In the same way, their evolution in the reference simulation will be considered to be indexed on the evolution of average gross wages, and not on prices as is the case in France since the 1993 reform. $\overline{RR}^{u,a}$ and \overline{RR}^{pr} are the gross replacement ratios of unemployment and pre-retirement benefits on the net of social contributions average wage of the age group (the seniors being taken as reference for the pre-retired):

$$b_t^a = \overline{RR}^{u,a} \cdot w_t^a \quad \text{for } a = j, m, v. \quad \text{And} \quad b_t^{pr} = \overline{RR}^{pr} \cdot w_t^v \quad (24)$$

The PAYG pensions system

The budgetary equilibrium of this system (eq. 25) is in line with the previous one (the subscription rate still being the adjustment variable), with the exception that unemployed and pre-retired people do subscribe:

$$\begin{aligned} \tau_t^r (w_t^j N_{t-1}^j + w_t^m N_{t-1}^m + w_t^v N_{t-1}^v + b_t^j U_{t-1}^j + b_t^v U_{t-1}^v + b_t^m U_{t-1}^m + b_t^{pr} PR_{t-1}^v) \\ = w_t^r (R_{t-1}^r + R_{t-1}^v) \end{aligned} \quad (25)$$

The gross replacement ratio \overline{RR}^r is defined relatively to the net average wage W_t in the economy.

$$b_t^r = \overline{RR}^r \cdot w_t^r \quad (26)$$

3.4. Calibration and dynamic simulation

It is possible using these equations to describe a simple dynamic of the labour markets with exogenously fixed wages. The multipliers can be eliminated by substituting values from eq. (19) in the arbitrage conditions given by eqs. ((20)-(21)). A linear function⁸ is chosen to describe the vacancies costs $C_t^a(V_t^a) = \bar{c}^a V_t^a$. The specific labour productivities are considered to increase by the exogenous constant rate g . Wages and marginal vacancies costs grow at the same rate. The following stationary equations can then be written (* indicates a stationarized variable):

$$\frac{1}{(1+g)\beta} \left(\frac{\bar{c}^{j*}}{q_t^j} - CF^{j*} \right) = A_0^j - \bar{w}_{t+1}^{j*} + (\bar{\mu}_t^j - \bar{\lambda}_t^j)(1 - \bar{\gamma}_t^j)(1 - \bar{\delta}_{t+1}^j) \left(\frac{\bar{c}^{j*}}{q_{t+1}^j} - CF^{j*} \right)$$

⁸In the annex, the results of an alternative simulation, using quadratic costs à la Bertola and Caballero (1994), are presented.

$$\begin{aligned}
 & + \bar{\lambda}_t^j (1 - \bar{\gamma}_t^m) (1 - \bar{\delta}_{t+1}^m) \left(\frac{\bar{c}^{m*}}{q_{t+1}^m} - CF^{m*} \right), \\
 \frac{1}{(1+g)\beta} \left(\frac{\bar{c}^{m*}}{q_t^m} - CF^{m*} \right) & = A_0^m - \bar{w}_{t+1}^{m*} + (\bar{\mu}_t^m - \bar{\lambda}_t^m) (1 - \bar{\gamma}_t^m) (1 - \bar{\delta}_{t+1}^m) \left(\frac{\bar{c}^{m*}}{q_{t+1}^m} - CF^{m*} \right) \\
 & + \bar{\lambda}_t^m (1 - \bar{\gamma}_t^v) (1 - \bar{\delta}_{t+1}^v) \left(\frac{\bar{c}^{v*}}{q_{t+1}^v} - CF^{v*} \right), \\
 \frac{1}{(1+g)\beta} \left(\frac{\bar{c}^{v*}}{q_t^v} - CF^{v*} \right) & = A_0^v - \bar{w}_{t+1}^{v*} \\
 & + (\bar{\mu}_t^v - \bar{\lambda}_t^v) (1 - \bar{\gamma}_t^v - \bar{\zeta}_t^{pr} - \bar{\zeta}_t^r) (1 - \bar{\delta}_{t+1}^v) \left(\frac{\bar{c}^{v*}}{q_{t+1}^v} - CF^{v*} \right).
 \end{aligned}$$

For the calibration procedure, there are now six additional equations, (13)–(15) and (20)–(22), for six new endogenous variables V^a and p^a ($a = j, m, v$). It is necessary to let some parameters adjust freely so as to reproduce the initial state of the labour market, notably the transition probabilities p^a given in table 3 (in this case, the number of successful matches during the initial period for each age group can be derived as the unemployment rate is known). The procedure used consists in, first, normalizing the equations by setting the average initial wage of middle aged workers at 100. Then with the above constraint, we adjust the average labour productivity (\bar{A}_0^a), the initial wages of young and senior workers (w_0^j and w_0^v), the scale parameters of the costs (\bar{c}^a) and matching functions (\bar{A}_M^a), so as to reproduce: 1.) the ratio md_0^j of the average productivity to the average wage for a young worker relative to the ratio of the average productivity to the average wage for a middle aged worker estimated by Crépon, Deniau and Pérez-Duarte (2002), 2.) the ratio md_0^v of the average productivity to the average wage for a senior worker relative to the ratio of the average productivity to the average wage for a middle aged worker estimated by the same authors, 3.) the ratio of the average productivity of a young worker to the average productivity of a middle aged worker derived from a quadratic equation describing the temporal profile of human capital as estimated by Welch (1979), 4.) the ratio of the average productivity of a senior worker to the average productivity of a middle aged worker (same procedure), 5.) the probabilities q^a , 6.) a mark-up ratio of average productivity to wage of 15% for middle aged workers.

In a matching model, it is necessary to have the values of the parameters governing the creation and destruction of vacancies. There is however a lack of data on those values which therefore have to be calibrated. To do so, hiring costs are supposed to be proportional (coefficient of 0.6) to wages (they so increase with age) and to be lower for senior workers (ad-hoc hypothesis of 0.15). The yearly discount rate is set at 5% and the yearly productivity growth at 1.25%. Using the above stationary equations, it is possible to evaluate the age specific quarterly probabilities of a vacancy being filled. That procedure gives low estimates for those parameters, which in turn leads to a very high number of vacancies. This has been corrected in previous studies by setting a higher discount rate, by reducing the mark-up ratio, by increasing search and hiring costs or adding training costs. This last solution is used here: those training costs are supposed proportional to wages and their intensity decrease with age (coefficients of

1.5, 0.75 and 0.05).

Unemployment and pre-retirement benefits are funded by workers contributions (in the facts, *DRE* beneficiaries are paid by the pensions system). *Unedic* data give an average net "general" replacement rate of 68% (*AUD* benefit) in 1999. However the share of unemployed workers, benefiting from this scheme, depends on age (44% of the young, 66% for middle-aged). At the same time, *ASS* (kind of a minimum unemployment benefit) and the minimum general life income (*RMI*) complement that system; the average net replacement rate for *ASS* is 43% and his average benefits amount is very close to the *RMI*. Those data, and the hypothesis that every unemployed is covered by one of those benefits schemes, enable the calculation of average age-specific net replacement rates of 54% and 60% for the young and middle aged categories. This rate is set at 68% for senior unemployed which benefit from numerous specific schemes. The average replacement rate for the various pre-retirement schemes is close to 66%. The budget equilibrium constraint (23) of the unemployment insurance system then gives a subscription rate of 5%.

Table 4: Model 1. Calibration

Fixed Parameters					
ϕ^j	=	0.5	ϕ^m	=	0.5
β	=	0.9879	g	=	0.0031
md_0^j	=	1.15	md_0^v	=	0.9
$\frac{\bar{A}_0^v}{\bar{A}_0^m}$	=	0.9725	$\frac{\bar{A}_0^m}{\bar{w}_0^m}$	=	1.15
$\overline{RRN}^{u,j}$	=	0.54	$\overline{RRN}^{u,m}$	=	0.6
			ϕ^v	=	0.5
			\overline{RRN}^{pr}	=	0.66
			$\frac{\bar{A}_0^j}{\bar{A}_0^m}$	=	0.9424
			$\frac{\text{pensions}}{\text{GDP}}$	=	0.125
			$\overline{RRN}^{u,v}$	=	0.68
Calculated Parameters					
\bar{A}_0^m	=	115	\bar{A}_0^j	=	108.4
\bar{c}^m	=	49.1	\bar{c}^j	=	60
\bar{A}_M^m	=	0.239	\bar{A}_M^j	=	0.3162
\bar{w}_0^j	=	81.95	\bar{w}_0^v	=	108.1
$\bar{\tau}_0^u$	=	0.058	$\bar{\tau}_0^r$	=	0.1375
			\bar{A}_0^v	=	111.8
			\bar{c}^v	=	16.2
			\bar{A}_M^v	=	0.1826
			\overline{RR}^r	=	0.48

For the retirement contribution rate, the calibration procedure is somewhat different: in 2000, retired received 12.1% of GDP as transfers. As the wages share in value added is around 2/3, the subscription rate required to ensure the budgetary balance of the system is 17.71%. This rate is considered to be the same for both workers and unemployed. It is then possible, knowing the size of the respective populations, to measure the replacement rate, defined as the average revenue of a retired relatively to the average revenue of a member of the active population. The value found is close to 50%. Let's note that, under those hypotheses, retired people over 65 have the same benefits than retired between 55 and 64. This can seem surprising as it assumes that pensions at retirement date are not higher than the average pension; however, figures

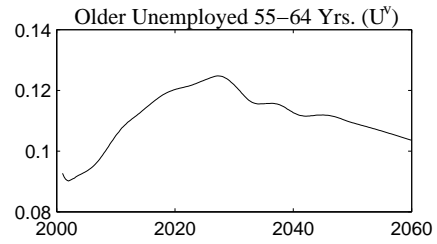
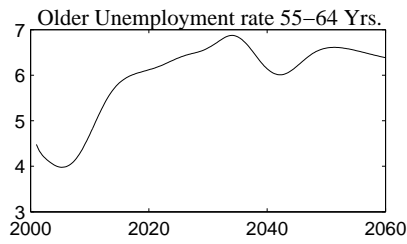
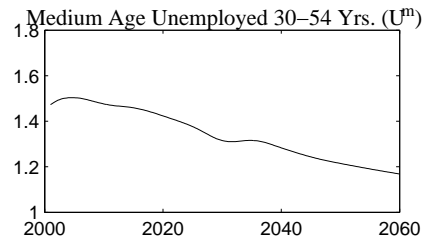
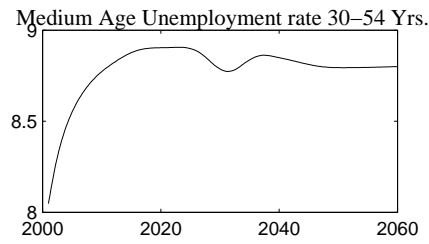
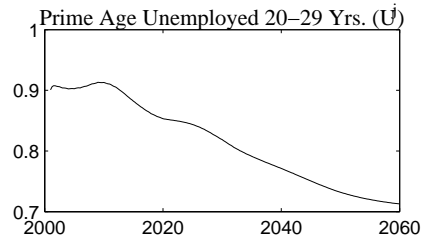
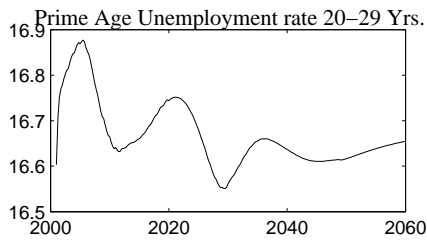
from the various pensions systems show that, because of the changes of indexation rules made in 1993, we are in line with this hypothesis at least for the reference year: in 1999, the average *CNAV* pension when cleared amounted to 480 € a month (compared to an average benefit of 500 €). In the same way, *ARRCO* and *AGIRC* figures were respectively 240 € (compared to 235) and 655 € (vs 720). Only civil service differed (1695 € versus 1575).

The simulation concludes that the global number of unemployed will decrease from 2.5 millions to 1.8 over the next 50 years (Figure 4). However, as the active population also strongly decreases, these evolutions will not entail a decrease of the unemployment rates: this rate increases for middle-aged and senior workers, and is rather stable for the young. As can be seen from those charts, the three labour markets are linked: the positive evolution of the middle aged unemployment rate between 2030 and 2040 does appear for senior workers around ten years later. Pure demographic composition effects largely dominate economic effects (as indicated by expected job values and quarterly transition probabilities which do not change by more than 1%, see figure 5). The impact on unemployment of those last effects has a magnitude order not over 0.1%. Vacancies profiles also follow closely the evolutions of active populations over the period. Flows variables (specifically the endogenous probabilities of evolution) jump quickly towards their equilibrium value, whereas the stocks variables adjustment takes some periods to complete.

Figure 4: Quarterly Unemployment by Age-Groups : Model 1.

Unemployment rate (percentages)

Unemployed (millions)



4. MODEL 2: ENDOGENOUS UNEMPLOYMENT WITH MATCHING AND WAGE BARGAINING

So as to fully close the model, it is necessary to determine the wage required by active people (the choice between activity and inactivity is still exogenous). In other words, the wage setting process needs to be spelled out.

4.1. Model improvements

Households behaviour

Let's note EN^a and EU^a the expected utilities of respectively having a job (and earning a wage) and of being unemployed (seeking a job and earning the benefits), then standard first order conditions are given by :

$$\begin{aligned} \frac{EN_t^{j^*}}{\beta(1+g)} &= (1 - \tau_t^u - \tau_t^r)\bar{w}_{t+1}^{j^*} + \lambda_t^j(1 - \gamma_t^m)[(1 - \bar{\delta}_{t+1}^m)EN_{t+1}^{m^*} + \bar{\delta}_{t+1}^m EU_{t+1}^{m^*}] \\ &+ (\mu_t^j - \lambda_t^j)(1 - \gamma_t^j)[(1 - \bar{\delta}_{t+1}^j)EN_{t+1}^{j^*} + \bar{\delta}_{t+1}^j EU_{t+1}^{j^*}] \end{aligned} \quad (27)$$

$$\begin{aligned} \frac{EU_t^{j^*}}{\beta(1+g)} &= (1 - \tau_t^u - \tau_t^r)\bar{b}_{t+1}^{j^*} + \lambda_t^j(1 - \gamma_t^m)[(1 - p_{t+1}^m)EU_{t+1}^{m^*} + p_{t+1}^m EN_{t+1}^{m^*}] \\ &+ (\mu_t^j - \lambda_t^j)(1 - \gamma_t^j)[(1 - p_{t+1}^j)EU_{t+1}^{j^*} + p_{t+1}^j EN_{t+1}^{j^*}] \end{aligned} \quad (28)$$

$$\begin{aligned} \frac{EN_t^{m^*}}{\beta(1+g)} &= (1 - \tau_t^u - \tau_t^r)\bar{w}_{t+1}^{m^*} + \lambda_t^m(1 - \gamma_t^v)[(1 - \bar{\delta}_{t+1}^v)EN_{t+1}^{v^*} + \bar{\delta}_{t+1}^v EU_{t+1}^{v^*}] \\ &+ (\mu_t^m - \lambda_t^m)(1 - \gamma_t^m)[(1 - \bar{\delta}_{t+1}^m)EN_{t+1}^{m^*} + \bar{\delta}_{t+1}^m EU_{t+1}^{m^*}] \end{aligned} \quad (29)$$

$$\begin{aligned} \frac{EU_t^{m^*}}{\beta(1+g)} &= (1 - \tau_t^u - \tau_t^r)\bar{b}_{t+1}^{m^*} + \lambda_t^m(1 - \gamma_t^v)[(1 - p_{t+1}^v)EU_{t+1}^{v^*} + p_{t+1}^v EN_{t+1}^{v^*}] \\ &+ (\mu_t^m - \lambda_t^m)(1 - \gamma_t^m)[(1 - p_{t+1}^m)EU_{t+1}^{m^*} + p_{t+1}^m EN_{t+1}^{m^*}] \end{aligned} \quad (30)$$

$$\begin{aligned} \frac{EN_t^{v^*}}{\beta(1+g)} &= (1 - \tau_t^u - \tau_t^r)\bar{w}_{t+1}^{v^*} + \lambda_t^v EPR_{t+1}^{r^*} + (\mu_t^v - \lambda_t^v)(\zeta_t^r EPR_{t+1}^{v^*} + \zeta_t^{pr} EPR_{t+1}^{v^*}) \\ &+ (\mu_t^v - \lambda_t^v)(1 - \gamma_t^v - \zeta_t^r - \zeta_t^{pr})[(1 - \bar{\delta}_{t+1}^v)EN_{t+1}^{v^*} + \bar{\delta}_{t+1}^v EU_{t+1}^{v^*}] \end{aligned} \quad (31)$$

$$\begin{aligned} \frac{EU_t^{v^*}}{\beta(1+g)} &= (1 - \tau_t^u - \tau_t^r)\bar{b}_{t+1}^{v^*} + \lambda_t^v EPR_{t+1}^{r^*} + (\mu_t^v - \lambda_t^v)(\zeta_t^r EPR_{t+1}^{v^*} + \zeta_t^{pr} EPR_{t+1}^{v^*}) \\ &+ (\mu_t^v - \lambda_t^v)(1 - \gamma_t^v - \zeta_t^r - \zeta_t^{pr})[(1 - p_{t+1}^v)EU_{t+1}^{v^*} + p_{t+1}^v EN_{t+1}^{v^*}] \end{aligned} \quad (32)$$

$$\begin{aligned} \frac{EPR_t^{v^*}}{\beta(1+g)} &= (1 - \tau_t^u - \tau_t^r)\bar{b}_{t+1}^{pr^*} + (\mu_t^v - \lambda_t^v)[\bar{\zeta}_t EPR_{t+1}^{v^*} + (1 - \bar{\zeta}_t)EPR_{t+1}^{v^*}] \\ &+ \lambda_t^v EPR_{t+1}^{r^*} \end{aligned} \quad (33)$$

$$\frac{EPR_t^{r^*}}{\beta(1+g)} = w_{t+1}^{r^*} + (\mu_t^v - \lambda_t^v)EPR_{t+1}^{v^*} + \lambda_t^v EPR_{t+1}^{r^*} \quad (34)$$

$$\frac{EPR_t^{r^*}}{\beta(1+g)} = w_{t+1}^{r^*} + \mu_t^r EPR_{t+1}^{r^*} \quad (35)$$

With τ_t^u and τ_t^r the unemployment and retirement contribution rates on wages, b^a the average unemployment benefit of an unemployed individual of age group a , b^{pr} the pre-retirement allowance and b^r the retirement pension.

Wage bargaining process and outcome

The negotiation between a worker, whether potentially or already employed, and a firm aims at setting a wage level so as to share the global surplus generated by the successful match. The standard formulation supposes the absence of global co-ordination at the aggregate level: the negotiation is a purely bilateral process between the firm and the worker (the effective labour volume being set in the end by the firm following *right to manage* hypothesis). The resulting Nash equilibrium is non co-operative as third agents actions are taken as given during the bargaining. The objective of each participant in the Nash criterion is to maximize its gains, i.e. the difference between the income when bargaining is successful and when it breaks down. It will be denoted Φ_t^{a*} for the firm (because here the fall-back position is simply equal to 0) and $(EN_t^{a*} - EU_t^{a*})$ for the worker. The prevailing wage contract at date t will be the solution of the following bargaining problem⁹ :

$$\max_{w^a} (EN_t^{a*} - EU_t^{a*})^{n^a} (\Phi_t^{a*})^{(1-n^a)}, n \in [0, 1]$$

where n^a is an age specific exogenous indicator of bargaining power. Given the Cobb-Douglas nature of the bargaining function, the first-order condition to this program can be re-written as the following rent-sharing rule:

$$(EN_t^{a*} - EU_t^{a*}) = n^a (EN_t^{a*} - EU_t^{a*} + \Phi_t^{a*}) \quad (36)$$

Now at the equilibrium, the macroeconomic levels of unemployment in the different cohorts have an influence on the "individual" negotiated wage through the probability of finding a job.

4.2. Base case : calibration and simulation

Compared to model 1, there are three new additional equations (36) linking wages and bargaining powers. Those age-specific bargaining powers adjust so as to keep the initial wages previously calibrated.

Table 5: Calculated Relative Bargaining Power of Salaries

$$\bar{n}^j = 0.195 \quad \bar{n}^m = 0.285 \quad \bar{n}^v = 0.8$$

A weighted average value of workers bargaining power of 0.31 is obtained, a figure in the range of the estimates of 0.2 by Cahuc, Gianella and Goux (1998) and 0.4 by

⁹As a simplification measure, it is supposed that newly hired workers do not support the hiring or training costs, leading to identical wages outcomes for new hires or renewals of jobs contracts.

Abowd and Allain (1996) on French data. As expected, bargaining power increases with age.

The evolutions in figure 5 sharply contrast with results presented in figure 4. The unemployment rate does increase for all three categories. This difference in results partly stems from the evolution of jobs expected values: while those values were quite stable in the previous simulation, they now decrease by more than 10% for each age group. The main cause of those evolutions lies in the doubling of pensions subscription rates. The very negative effect induced by the fiscal wedge in that kind of model can be discussed; however, the reason for the increase in this rate are genuinely demographic if the relative purchasing power of pensioners is to be kept steady (cf the mechanic increase in the relative number of retired to active people emphasized in all reports on pensions systems future (Charpin 1999)). The increase in mandatory contribution rates entails a rise in unemployment. At the stationary equilibrium, this impact depends on the degree of proportionality between unemployed and workers earnings. Here this proportionality is absolute as unemployed face the same subscription rate as workers, and as there is no loss in utility associated to labour. Taking into account such elements will push-up the increase in unemployment. It is to note that in a standard matching labour market model, such a proportionality leads to independence in the long run equilibrium between unemployment and the tax system. The model presented here differs as the unemployment replacement rate is age-dependent and as, more importantly, workers can enter retirement. Pensioners earnings enter symmetrically in the expected utilities of both workers and unemployed: the proportionality in actual incomes does not entail a proportionality in expected utility! In that configuration, fiscal wedge mechanisms have an impact on the unemployment rate at the stationary equilibrium. The slight increase in gross wages (corrected by productivity growth) does not balance the rise in taxes, leading to a decline in the relative financial position of the active people.

Another important difference to the previous model lies in the evolution of the age specific transition probabilities: job finding probabilities strongly decrease, raising fears of a very segmented labour market (with all the problems caused by long spells of unemployment). The rise in the probability of a vacancy being filled via a successful matching is not important enough to balance the decrease in the number of vacancies offered. Given the perfect substitutability between workers in the model, manpower shortage kind-like effects don't appear.

Assuming quadratic vacancy costs does not change the sense of the evolutions but does limit the previously observed impacts: the rise in unemployment in young decreases from 3% to 1% (Fig. 10 in the appendix). Those differences in results stem from the different evolutions of the probabilities of finding a job or filling a vacancy. Alongside the study of various *scenarii* under alternative closure rules (see next part), some sensitivity analysis were done. The results are given in the annex (Table 6). As soon as the calibration procedure is run again, the model results appear to be fairly robust. Surprisingly, modifications of the growth rate, the initial transition probabilities or unemployment rates, and of the hiring and training costs have a limited impact. Only changes in wage seniority rules, relative bargaining powers or in the initial age specific

productivity do entail substantial evolutions.

Figure 5: Quarterly Unemployment by Age-Groups : Model 2.

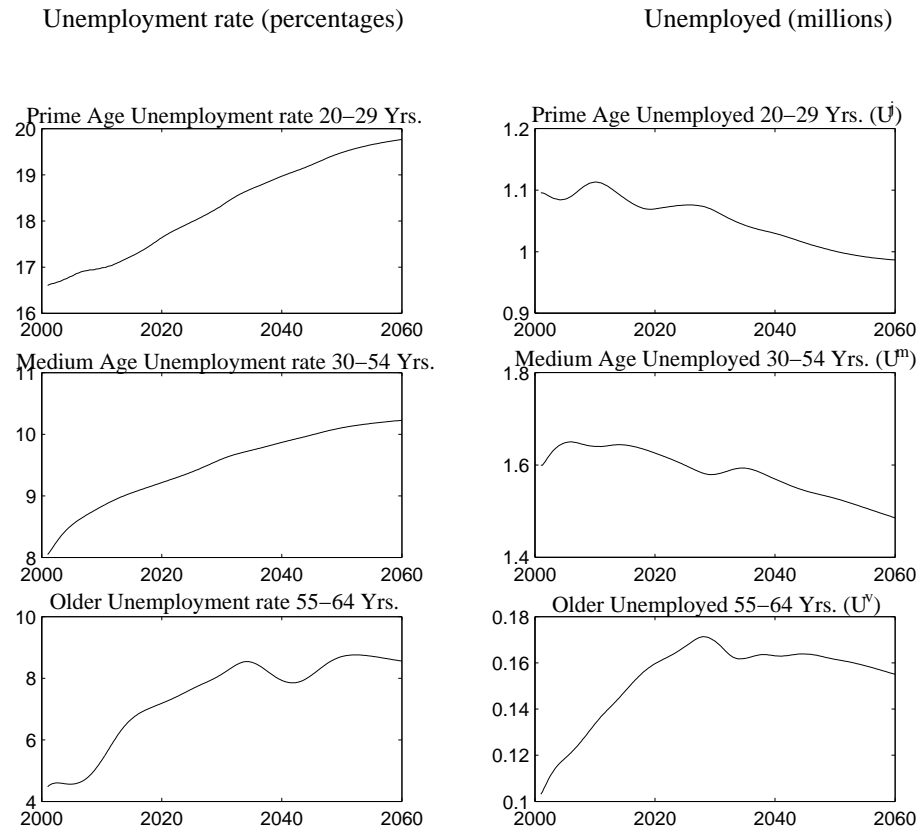
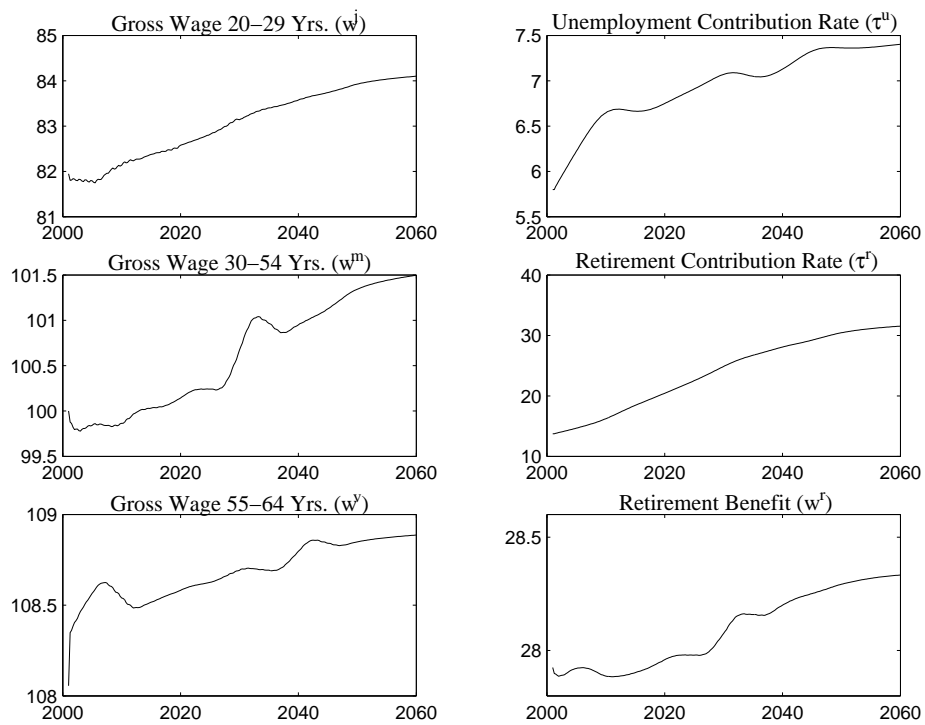


Figure 6: Gross earnings and payroll tax rates



4.3. Alternative reforms of the pensions system

The reference scenario underlines the problems raised by maintaining the status-quo on pensions rules. So as to look at the available room for manoeuvre, two "extreme" scenarios are studied. Their conclusions need to be handled with care but give interesting orders of magnitude: part of the optimal solution to enable the French PAYG system to survive is doubtless a mix of those *scenarii*.

Constant Retirement Contribution Rate

In the first *scenario*, the contributive rate to retirement is set invariant, equal to its initial value. The adjustment variable is now the pensions replacement rate. Results of this simulation are given in Figure 7. The former rise in unemployment becomes limited (results are on par with those of Model 1), but the price to pay lies in a strong increase of intergenerational inequalities. The replacement rate drops by more than a half.

Gradual postponement of retirement age

The second *scenario* looks at an increase in working (and subscription) legal duration. The mandatory retirement age goes up to 65 years. This is first modelled by 1.) shifting the probability ζ^{pr} of entering pre-retirement to zero just after the initial date (those schemes become no longer possible from the 2nd quarter of 2001); 2.) lessening gradually to zero the probability ζ^r for a senior worker of entering retirement. This probability is reduced linearly from and reaches zero in 2050¹⁰. As in the reference scenario, the retirement contribution rate is the variable of adjustment. As was expected, those hypotheses strongly limit the rise in young and middle-aged unemployment rates (Fig. 8). However, they are not sufficient enough to stop the rise in subscription rate necessary to ensure the budgetary equilibrium of the PAYG system: the tax pressure still increases from 14% to 23% in 2035, before easing a little. The unemployment rate decreases for the seniors. This result stems from the non modification of the parameters driving seniors unemployment. This issue was addressed by considering that people formerly entering pre-retirement now become unemployed (the idea being that pre-retirement schemes were used as a way to mask the seniors unemployment problem). Technically, after the initial date the probability of becoming unemployed will be now adjusted in order to take into account the previous probability of entering pre-retirement. As expected (Fig 9), the seniors unemployment rate very strongly increases (of course, one can argue, the probability of leaving unemployment should also have been modified).

¹⁰Initially the reform is unexpected but once it is announced in 2001 the full path of $\{\zeta_t^r\}_{t>1}$ is perfectly anticipated by the agents.

Figure 7: Quarterly Unemployment by Age-Groups : Constant Retirement Contribution Rate

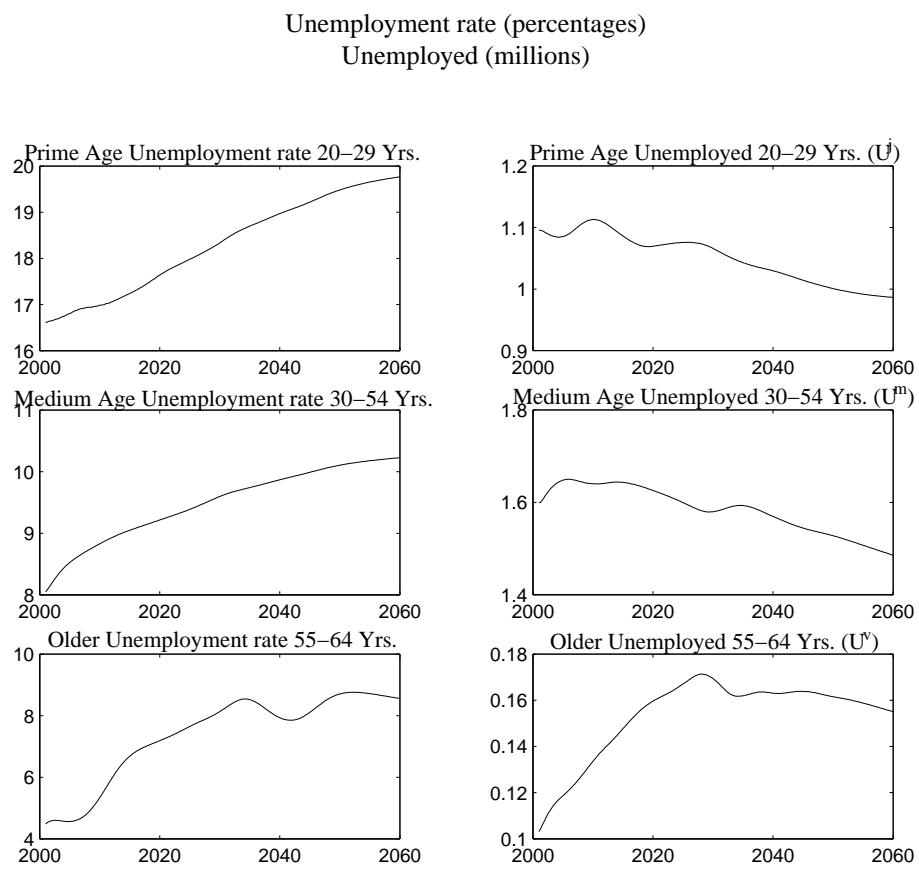


Figure 8: Quarterly Unemployment by Age-Groups: 5 Yrs.-gradual postponement of retirement age

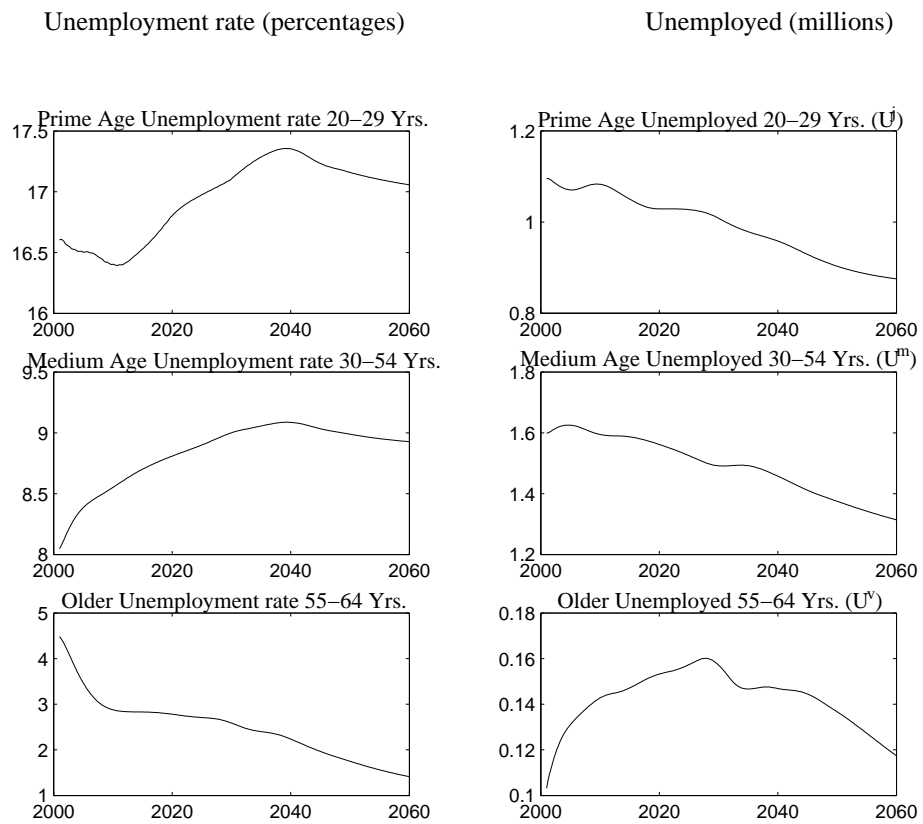
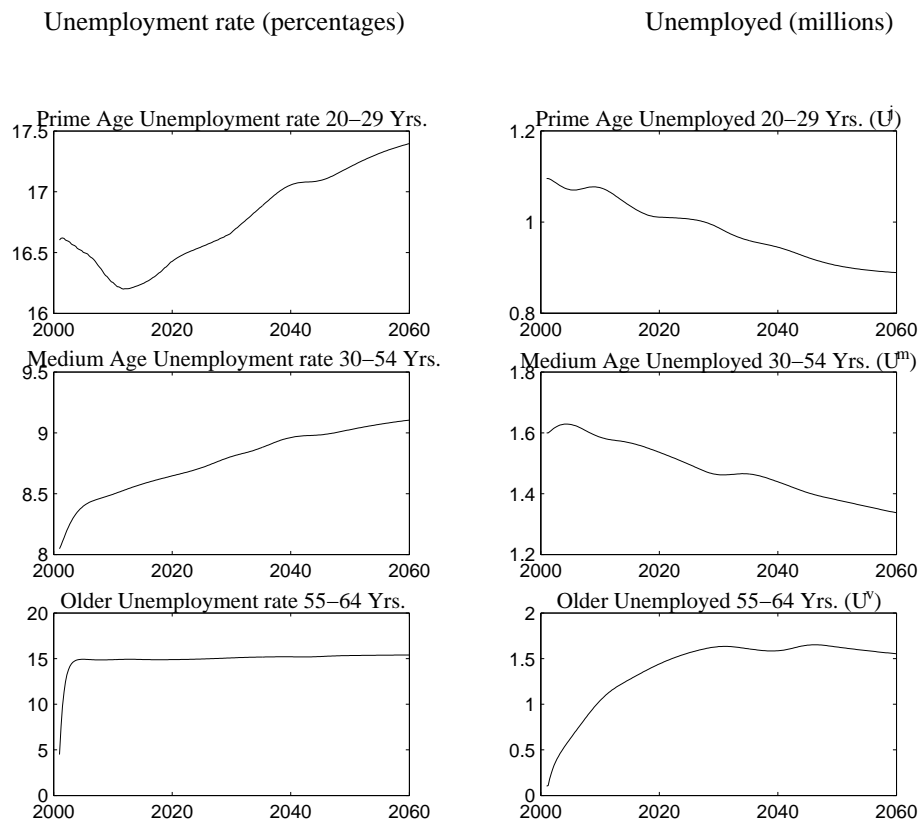


Figure 9: Quarterly Unemployment by Age-Groups : 5 Yrs.-gradual postponement of retirement age with δ^v adjustment



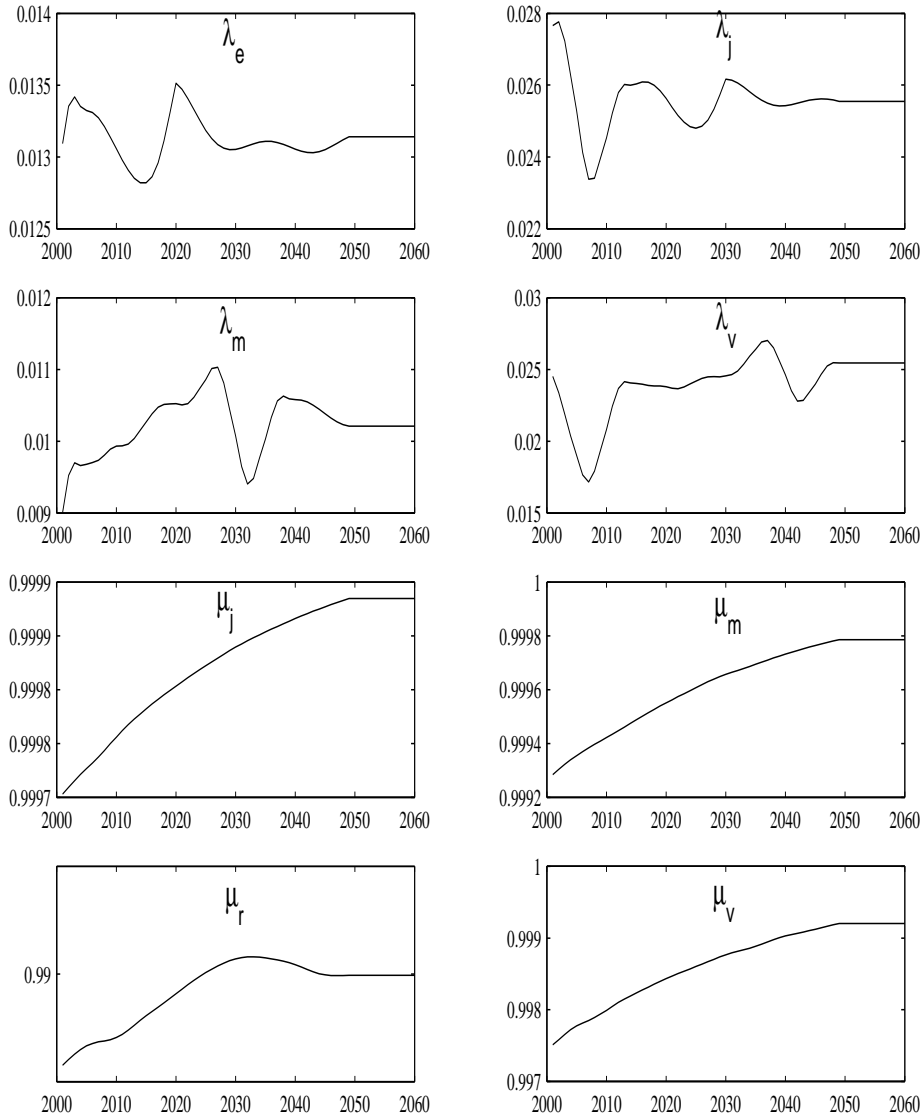
5. CONCLUSION

Numerous studies have looked at the potential impacts of the forthcoming demographic evolution on the labour markets. In sharp contrast with the standard wisdom according to which the decrease in the labour supply will entail a decrease in unemployment (to a degree depending on wage rigidities), the matching model used here and applied to French data stresses the problems linked to those evolutions and the importance of pensions closure rules. The doubling of the ratio pensioners on active population in 50 years would lead in the absence of reforms to an increase in subscription rates and so of the fiscal wedge; this would entail a rise of unemployment. At the same time, keeping the subscription rates at their initial level would lead to a serious decline in the relative income position for the old people. An increase in mandatory work duration is an interesting way to mitigate those effects; such a disposition is however strongly resisted against, and it would be needed, for this reform to be successful, to see a change in firms behaviour towards senior workers.

The work presented in this paper can of course be improved upon as the rules governing the labour market are taken as invariant. So no endogenous self-correction mechanism is considered; however it is highly likely that parameters such as activity choice's determinants, wages seniority indicators or mandatory annual work length will adapt in answer to demographic changes. This work is just a first step towards a more detailed description of these issues. At this stage, some research paths appear worthwhile : endogenous leisure activity trade-off; testing various degrees of labour markets segmentation and of cohort effects *via* for example the use of an explicit production function; looking more closely at fiscal wedge kind of effects; better detailing of the age profiles of productivity (generation and seniority effects); refining the activity cessation decision. We expect that taking into account such effects would help designing better reforms aiming at preserving the French PAYG pension system while keeping in mind at the same time equity considerations.

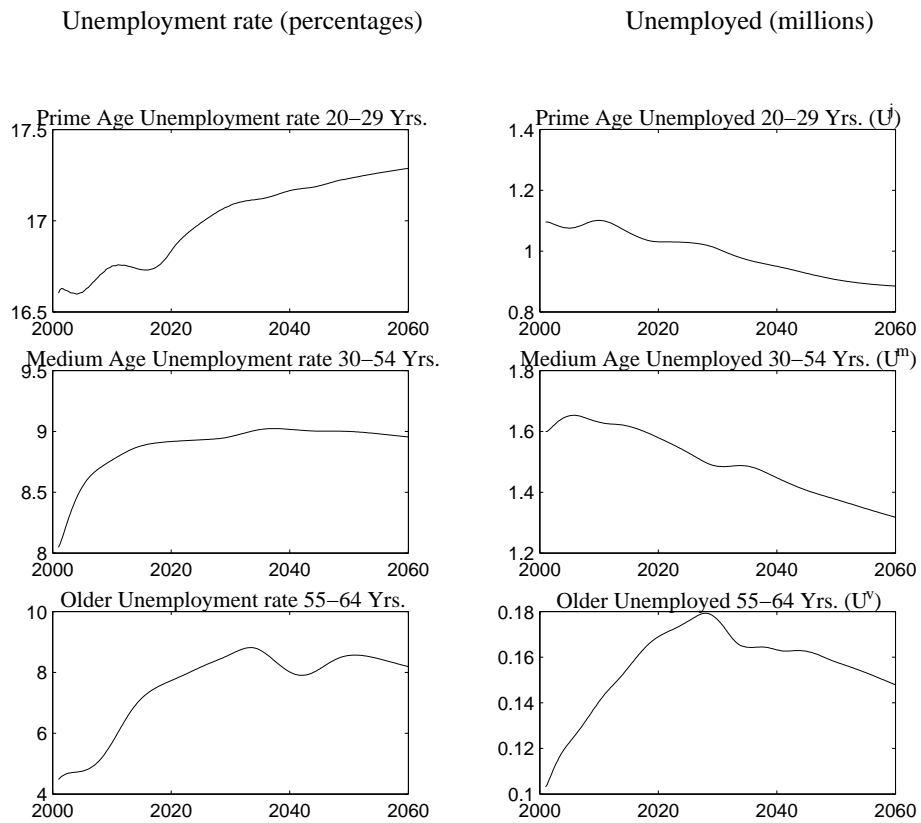
A APPENDIX

A1. Exogenous quarterly probabilities of death and ageing



A2. Model 2. with Quadratic Vacancies Costs

Figure 10: Quarterly Unemployment by Age-Groups



A3. Sensitivity Analysis for Model 2.

Table 6: Final Steady-State Comparisons (percentage change to initial value)

	base case	10 percent increase in					$md^a = 0$ $A_0^a = A_0^m$
		t^e	g	p_0^a	u_0^a	c_0^a	
τ^r	+112.5	+111.7	+112.5	+112.5	+112.8	+112.5	+113
τ^u	+25.82	+25.40	+25.79	+25.31	+25.73	+25.82	+29.07
EN^j	-19.84	-19.69	-19.86	-19.90	-20.17	-19.84	-20.58
EN^m	-19.04	-18.91	-19.09	-19.02	-19.25	-19.04	-18.86
EN^v	-11.67	-11.58	-12.01	-11.66	-11.80	-11.67	-10.56
EU^j	-20.11	-19.96	-20.12	-20.16	-20.45	-20.11	-21.00
EU^m	-19.41	-19.27	-19.46	-19.37	-19.63	-19.41	-19.22
EU^v	-11.78	-11.69	-12.15	-11.76	-11.91	-11.78	-10.62
p^j	-14.75	-14.60	-14.72	-15.40	-15.75	-14.75	-21.71
p^m	-12.65	-12.56	-12.63	-12.71	-12.94	-12.65	-12.50
p^v	-12.34	-12.23	-12.33	-12.34	-12.52	-12.34	-10.21
u^j	+16.28	+15.55	+16.24	+17.27	+16.93	+16.28	+25.87
u^m	+24.84	+24.45	+24.81	+23.74	+24.02	+24.84	+25.08
u^v	+68.44	+68.86	+68.40	+63.85	+64.59	+68.44	+65.53
U^j	-12.99	-13.07	-13.02	-12.39	-12.34	-12.99	-7.13
U^m	-17.71	-17.87	-17.72	-18.34	-18.26	-17.71	-17.46
U^v	+22.38	+22.46	+22.35	+19.52	+19.72	+22.38	+20.65
V^j	-36.77	-36.60	-36.75	-37.30	-37.77	-36.77	-43.08
V^m	-37.22	-37.20	-37.20	-37.79	-38.04	-37.22	-36.80
V^v	-5.97	-5.66	-5.96	-8.17	-8.38	-5.97	-2.73
Φ^j	-8.02	-7.96	-8.03	-7.61	-7.77	-8.02	-4.22
Φ^m	-9.75	-9.68	-9.76	-9.63	-9.80	-9.75	-9.81
Φ^v	-10.56	-10.46	-10.56	-10.55	-10.70	-10.56	-9.87
w^j	+2.29	+2.27	+2.29	+2.11	+2.16	+2.29	-0.17
w^m	+1.30	+1.29	+1.30	+1.29	+1.32	+1.30	+1.42
w^v	+0.71	+0.71	+0.71	+0.71	+0.71	+0.71	+2.95
w^r	+0.99	+0.95	+0.99	+0.97	+1.00	+0.99	+1.21

References

- Abowd, J. and L. Allain, Compensation Structure and Product Market Competition, *Annales d'Economie et de Statistiques*, 1996, 41/42, 207–217.
- Aglietta, M., D. Blanchet, and F. Héran, *Démographie et Economie*, la documentation française editor, Vol. 35 of *Rapport au Premier Ministre*, Paris: Conseil d'Analyse Economique, Février 2002.
- Bertola, G. and R. Caballero, Cross-Sectional Efficiency and Labour Hoarding in a Matching Model of Unemployment, *Review of Economic Studies*, July 1994, 61 (3), 435–456.
- Blanchard, O. and P. Diamond, The Aggregate Matching Function, in P. Diamond, editor, *Growth Productivity Unemployment*, Cambridge MA: MIT Press, 1990, chapter 9, pp. 159–201.
- Blanchet, D., *L'impact des changements démographiques sur la croissance et le marché du travail : faits, théories et incertitudes*, mimeo, INSEE, Malakoff Mars 2001.
- Brondel, D., D. Guillemot, L. Lincot, and P. Marioni, La population active devrait encore augmenter pendant une dizaine d'années, *Economie et statistiques*, 1996, 300, 13–38.
- Cadiou, L., J. Genet, and J.L. Guérin, *Evolution démographique et emploi*, Mimeo, CEPPII, Paris Mars 2000.
- Cahuc, P. and A. Zylberberg, *Economie du travail* Balises, de boeck université editor 1996.
- , C. Gianella, and A. Goux D. ans Zylberberg, *Equalizing Wage Differences and Bargaining Power*, Working Paper G9805, Insee, Malakoff Juin 1998.
- Charpin, J., *L'avenir de nos retraites* Rapports Officiels, la documentation française editor, Paris: Commissariat Général du Plan, 2 trimestre 1999.
- Cnav, *Recueil Statistique*, Publication Cnav, Novembre 2001.
- COR, *Retraites : renouveler le contrat social entre les générations : Orientations et débats.*, la documentation française editor, Vol. 1 of *Rapport au Premier Ministre*, Paris: Conseil d'orientation des retraites, Décembre 2001.
- Crépon, B., N. Deniau, and S. Pérez-Duarte, *Wages, Productivity, and Worker Characteristics : A French Perspective*, Mimeo, CREST, INSEE Janvier 2002.
- Destinie, Equipe, *Le modèle de microsimulation dynamique*, Working Paper G9913, Insee, Malakoff Aout 1999.
- Domingues Dos Santos, M., Vieillesse démographique et chômage, *Revue française d'économie*, Juillet 2001, 16 (1), 3–50.
- Gruber, J. and D.A. Wise, *Social Security and Retirement around the world* Conference Report, university of chicago press editor, NBER, 1999.
- Layard, R., S. Nickell, and R. Jackman, *Unemployment, Macroeconomic Performance and the Labour Market*, Oxford University Press, 1991.
- Meslé, F. and J. Vallin, *tables de mortalité Française pour les XIX et XX siècle et projections pour le XXI siècle*, Technical Report, INED, Paris 2001.

- Mesnard, O., Les retraites en 2000, *DREES : Etudes et Résultats*, Décembre 2001, 147, 1–12.
- Mortensen, D. and C. Pissarides, Job Reallocation, Employment Fluctuations and Unemployment, in J. Taylor and M. Woodford, editors, *Handbook of Macroeconomics*, north-holland editor, Vol. 1, Amsterdam: Elsevier, 1999, chapter 18, pp. 1171–1228.
- Nickell, S. and S. Andrews, unions, real wages and unemployment 1951-79, *Oxford Economic Papers*, 1983, 35 (supplement), 183–206.
- Pissarides, C., *Equilibrium Unemployment Theory*, Basil Blackwell, 1990.
- Quintreau, B., *Age et emploi à l'horizon 2010*, journal officiel editor, Vol. 20 of *Rapport*, Paris: Conseil Economique et Social, Octobre 2001.
- Stock, JH. and D.A. Wise, The Option Value of Work and Retirement, *Econometrica*, September 1990, 58 (5), 1151–80.
- Welch, F., Effects of Cohort Size on Earnings, *Journal of Political Economy*, October 1979, 87.