Ageing and Unemployment : A matching model prototype

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Extremely preliminary draft

1 Introduction

The ageing process in the developed countries yet has and will probably have important implications on the labour markets. As an example, let's recall that, in France, until 2006, the 20-59s and their share within the overall population will increase to 54 %. Between 2006 and 2020, the size of this category will drop by 0.2 % per year. Between 2020 and 2035, this drop will accelerate to reach 0.3 % per year. It will go back to 0.2 % after 2050. In 2050, the share of this category will represent 45 % of the overall population. This phenomenon represents an important shift if we compare to the situation that was prevailing before 1995: the share of the category (20-59s) was nearly constant, moving between 51 and 55 %. This decrease in this pivot generation share is the main source of the present anxiousness (for instance, for the pension schemes equilibrium, the dependency ratio i.e. the ratio of the 65 and over to the 20-64s will increase from 24 % in the 90s to 36.5 % in 2020 before reaching 55 % in 2020). In addition, the structure of the ageing population (18-64s) will change. The median age, that was 40 in 1950, 37 in 1980 and is 39 today, will reach 42 in 2015.

The first usual question is about the ability of the slow down of the working age population to lower the unemployment rate; a second question concerns the implied changes in the wages formation: will the wage setting according to qualifications and experiences change with the ageing process? Another question deals with the situations within the labour market. The latter question is the main purpose of this paper; in which we study the impact of

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the ageing process on the wage earners' situation when they are differentiated according to their age.

The study of the links between the ageing process and the wage earners' situations would require a decomposition of the labour force according to the age and the qualifications. We focus here on the age decomposition in order to study later the impact of the retirement pension schemes reforms due to the present and forthcoming financial disequilibria that will probably imply new rules for the pre retirement schemes. The self regulations of the labour markets (such as participation rates increase, changes in the wage setting, increasing in the retirement ages, increasing in the activity rates for the eldest workers ...) have to be studied in order to know if they are able to neutralize the effects of the decrease in the working age populations.

Artus [2001], uses a production function with two classes of substitutable workers: qualified and non qualified. He shows that the existence of qualified workers (whose market is a competitive one) lowers the non qualified negotiated real wages. The reason is that the presence of qualified workers increases the elasticity of the firms work demand for non qualified workers with regard to the non qualified wages. This type of model is interesting; distinguishing the various types of workers and studying the different linked labour markets with a substitution assumption.

We adopt a close way, distinguishing three types of workers: young, adult, and old. These three categories are living together on the labour market and their sizes are varying. The relative sizes of the three categories depend of the pyramid of ages moves, for example the inflation of a part of the pyramid due to the ageing process.

We study the workers' supply behaviours within the labour markets and the equilibria. In this preliminary version, the demand from the firms is in line with the usual WS-PS models. We focus on the different supply behaviors from the different age categories according to their age specific alternative incomes: guaranteed minimum income for the youngest, pre-retirement pensions for the eldest. We observe the wages formation by age by adapting a matching model (Mortensen and Pissarides, 1999) and integrating the different alternative incomes that are provided by the welfare scheme. We do not study the human capital formation and its changes along the ages (which is the purpose of a paper as Polachek, 1995).

That means that we do not study the transitions towards pre-retirement according to the wage earners' individual stories (which is the case in Bommier et alii, 2001): in this paper the matching processes of the different age categories are not linked. On the other hand, old workers and firms do not match for pre-retirement vs employment. The firms' choice for pre-retirement is exogenous, the firms' behaviour does not take into account costs differences between pre-retirement schemes and lay- offs while these costs will be included in a further version.

This preliminary version deals with a theoretical exploratory model in which are mainly studied the answers of the labour markets differentiated by age to the present and forthcoming changes in the workers age pyramid. The model is presented in a first part

2 An age matching economy

2.1 Philosophy of the model

The framework used in this paper is a standard matching model along the lines of Diamond [1982], Mortensen [1982] and Pissarides [1990]. To take into account of generational heterogeneity we assume that there are separate labour market for different age worker. At a given time, an individual can only participate to its age-specific labour market. For simplicity, we assume that there is three homogenous labour market, each for some large groups of age : "young worker", "middle-aged worker" and "older worker". Individual like population ageing are in the heart of our concern so we assume that there is some possibility that an individual became older in the next period and then will may have to participate to an other age specific labour market in the future.

2.2 Notations

- The upper letter " ^{g} " on a variable indicates the age group. In this version of the paper $g = [y, m, o]$, where "y" is for "young worker", "m" is for "middle-aged worker" and "o" is for "older worker". Agents may also be retired "r" or early-retired "er".
- The active population N^g belonging to age group $g$is the sum of the unemployed people of age group u^g and the L^g employed people.
- Household Side
  - ρ^g is the positive and instantaneous rate of time preference of an age group (exogenous).
  - w^g is the instantaneous wage rate of a representative worker of group age g (endogenous).
  - b^g is the instantaneous income of a representative unemployed worker of group age g i.e. the reservation wage (exogenous here).

- U^g is the expected value of searching a job for an unemployed worker of group age g (endogenous).
- V^g is the expected value of holding a job for an employed worker of group age g (endogenous).
- R is the expected value of retirement (exogenous or endogenous).
- ϵ^o is the instantaneous probability of "being retired before the legal age" (early retirement) (exogenous or endogenous).
- T is the expected value of being early retired (endogenous).
- δ^g is the instantaneous probability of loosing a job for an employed people (exogenous).
- p^g is the instantaneous probability of finding a job for an unemployed people (endogenous).
- λ^g is the instantaneous probability of "becoming an older people", i.e. of belonging to the next group of age (exogenous). λ^r is the instantaneous probability of a retired worker to dying between periods (exogenous)
- μ^g is the instantaneous conditional probability of an individual to loose its job when it "become an older people" (exogenous ?).

• Firm Side

- E^g is the expected value of holding an age group g job vacant for a firm (endogenous).
- J^g is the expected value of filling a job for a firm (endogenous).
- a^g is the instantaneous value added obtained by a firm from an employed individual of group g (exogenous ?).
- c^g is the positive instantaneous cost of recruiting a worker to fill a vacancy (exogenous).

• β^g fraction of matching surplus received by the worker after Nash bargaining.

• θ^g age-g labour market tightness.

2.3 The model

Agents are of two types, workers and employers. The key assumption of our model is the heterogeneity of the labour force according to its age. Although individual belonging to a same age groups are perfectly homogenous and mobile, they can apply for any age-specific job. We consider identical individuals. Agents are making rational expectations.

For tractability we built some kind of aging process as follows. The economy is composed of numerous identical worker individual that belongs to different age group : young worker, middle-aged worker, old worker, early-retired and retired. The population of each group has different size.
Our key assumptions are : (1.) the behavior of all individuals belonging to a same given age-group are homogenous independently of its true age, (2.) the expected future life as a typical individual that belongs to a specific age-group is unknown and each individual of this group "g" face an identical and constant probability per unit time λ^g of being "older" (of belonging to the next upper age group at the next time).

2.3.1 Worker Euler Equations

The instantaneous utility function of each group-age individual is linear in consumption and future consumption is discounted at an exogenous constant rate ρ^g. Because we do not deal with financial aspects, agents only consume their current wage or pension income. Hence, group-age individuals act as expected present value maximizers.

Each age-g worker who is employed in the current period is paid the current net real age-specific-wage w^g. This worker has an age-specific probability δ^g to loose its job at the end of the period and then it will be unemployed (here this job destruction rate is exogenous). On the other side, an unemployed worker receives current benefits b^g, this generic variables embodied lot of things unemployed replacement income, utility of leisure, cost of searching, ... Unemployed worker has an instantaneous probability p^g to find a job for the next period. Let now comment the solution to the maximized behavior of young and medium age risk-neutral workers. Let V^g and U^g be respectively the current present value of utility of an age-g worker when it is actually employed and when it is actually unemployed. These are given by the following expressions ($'$ express the current expectation of the next period value) (1)–(4).

$$\rho^y V^y = w^y + \delta^y (U^{y'} - V^{y'}) + \lambda^y [(1-\mu^y)(V^{m'} - V^{y'}) + \mu^y (U^{m'} - V^{y'})] \quad (1)$$
$$\rho^y U^y = b^y + p^y (V^{y'} - U^{y'}) + \lambda^y (U^{m'} - U^{y'}) \quad (2)$$

$$\rho^m V^m = w^m + \delta^m(U^{m'} - V^{m'}) + \lambda^m[(1-\mu^m)(V^{o'} - V^{m'}) + \mu^m(U^{o'} - V^{m'})] \quad (3)$$
$$\rho^m U^m = b^m + p^m(V^{m'} - U^{m'}) + \lambda^m(U^{o'} - U^{m'}) \quad (4)$$

Notice that we assume that the probability of loosing a job may be influenced by the "aging" of the individual whereas the possibility of being reemployed for a current unemployed individual occur only in the current and same age-group. In other word μ^y is the instantaneous probability that a young worker loose its job when it become a medium age worker

$$\rho^o V^o = w^o + \delta^o(U^{o'} - V^{o'}) + \lambda^o(R' - V^{o'}) + \epsilon^o(T' - V^{o'}) \quad (5)$$
$$\rho^o U^o = b^o + p^o(V^{o'} - U^{o'}) + \lambda^o(R' - U^{o'}) + \eta^o(T' - U^{o'}) \quad (6)$$

A retired individual (8) is assumed to receive pension benefits e^o, which are given at this stage. He has an instantaneous probability μ^o of dying before the next period. Some individual may also have an opportunity of benefit to a pre-retirement scheme, in these case there discounted utility is given by (7), with d^o is the current pension allowed to early retired individuals. At this stage of the research nothing will really distinguish this opportunity from retirement.

$$\rho^o R = e^o + \lambda^r(0 - R') \quad (7)$$
$$\rho^o T = d^o + \lambda^{er}(R' - T') \quad (8)$$

2.3.2 Firm side

The average productivity a^g of a worker like its wage rate are assumed to be function of the group it belongs. $a^g - w^g$ is the current added value for the firm of an age g employed individual. Under the assumption of fixed capital and real interest rate, the expected present value of future profit of a firm on an age-specific worker is J^g :

$$\rho J^y = a^y - w^y + \delta^y(E^{y'} - J^{y'}) + \lambda^y[(1-\mu^y)(J^{m'} - J^{y'}) + \mu^y(0 - J^{y'})] \quad (9)$$
$$\rho J^m = a^m - w^m + \delta^m(E^m - J^{m'}) + \lambda^m[(1-\mu^m)(J^{o'} - J^{m'}) + \mu^m(0 - J^{m'})] \quad (10)$$
$$\rho J^o = a^o - w^o + \delta^o(E^{o'} - J^{o'}) + \lambda^o(0 - J^{o'}) + \epsilon^o(0 - J^{o'}) \quad (11)$$

At this stage we assume that firms do not suffer any costs from firing people.

Firms posting vacancies generate current loss c^g per vacancy and period (i.e. cost of recruiting a worker to fill a vacancy). Let denote q^g the frequency with wich the firm encounters worker seeking a job (see below) the expected value of holding an age-specific job vacant E^g :

$$\rho E^y = -c^y + q^y(J^{y'} - E^{y'}) \quad (12)$$
$$\rho E^m = -c^m + q^m(J^{m'} - E^{m'}) \quad (13)$$
$$\rho E^o = -c^o + q^o(J^{o'} - E^{o'}) \quad (14)$$

2.3.3 Nash Bargaining wage outcome

At the beginning of each period each firm and employee bargain over the current wage, a successful bargaining is supposed to hold for a single period. The literature use to assume that the the total value of a match (i.e. the value that will be lost if the two parties cannot agree $W^g = J^g + V^g$) is split between the worker (*insider*) and the firm trough a Nash Bargaining on wage. From rationality, the maximized criteria is the following weighted of net gains of the two bargainers (i.e difference between the gain when the bargaining will success and the loss when it breaks down) (Binmore and alli. [1986]) .

$$\max_{w^g} \ (V^g - U^g)^{\beta^g}(J^g - E^g)^{(1-\beta^g)} \ \ \forall g \quad (15)$$

Notice that the division of the match rents will only be insure with the wage rate, the level of employment will residually be fixed by the firm as in Nickell and Andrews [1983]. $\beta^g \in [0, 1)$ indicates the relative bargaining power of the insider. With this kind of objective at the equilibrium, the insider will get the fraction β^g of the match surplus $S^g = W^g - U^g - E^g$ (this will need to be positive in order to insure the match formation) :

$$V^g - U^g = \beta^g S^g \Longrightarrow V^g - U^g = \frac{\beta^g}{1-\beta^g} J^g - E^g \quad (16)$$

2.3.4 Matching process

For a new job, before bargaining takes place unemployed worker and employer with vacancy have to meet.

We assume that job search activity and recruiting activity can be related with some matching process in order to give some kind of meeting rate between unemployed worker and vacant jobs for a successful new job creation M. With u the unemployed worker and v the vacancies, we assume that the meeting process between these flows of labour is describe by the following matching function :

$$M^g = M^g(u^g, v^g) = F^g\sqrt{u^g v^g} \Longrightarrow p^g = \frac{M^g}{u^g} = F^g\sqrt{\frac{v^g}{u^g}} = F^g\sqrt{\theta^g} = q^g\theta^g \tag{17}$$

Where θ^g, the age specific vacancy to unemployment ratio, signals labour market tightness, as before $p^g(\theta^g)$ is the probability for an unemployed to finding a job, the ratio of the rate of successful meetings to unemployment rate (unemployment spell hazard). Knowing that by definition q^g is the the probability of fulfill a vacancy, the following identities about flow rates $\lambda u \equiv M \equiv qv$ always hold. The square formulation of these formulation is common in applied matching literature (Pissarides and Mortensen [1999]). Notice that matching and bargaining processes are intimately linked, an actually employed worker has some bargaining power because there is a cost of searching a new employee for the firm to fill its possible vacancy (i.e. E, U is the threat point for the employee in the bargaining process).

2.4 Stationary and Symmetric equilibrium

In the stationary equilibrium all values are constant and because the economy is non-stochastic we have $x' = x$, so the values of an employed worker and of an unemployed worker (1)–(8) give us after straight calculations (and assuming $\eta^o = \epsilon^o$) :

$$V^y = \frac{\tilde{\rho}_u^y[w^y + \lambda^y(1-\mu^y)V^m + \lambda^y\mu^y U^m] + \delta^m[b^y + \lambda^y U^m]}{\tilde{\rho}_v^y\tilde{\rho}_u^y - \delta^y p^y} \tag{18}$$

$$U^y = \frac{p^y[w^y + \lambda^y(1-\mu^y)V^m + \lambda^y\mu^y U^m] + \tilde{\rho}_v^y[b^y + \lambda^y U^m]}{\tilde{\rho}_v^y\tilde{\rho}_u^y - \delta^y p^y} \tag{19}$$

$$V^m = \frac{\tilde{\rho}_u^m[w^m + \lambda^m(1-\mu^m)V^o + \lambda^m\mu^m U^o] + \delta^m[b^m + \lambda^m U^o]}{\tilde{\rho}_v^m\tilde{\rho}_u^m - \delta^m p^m} \tag{20}$$

$$U^m = \frac{p^m[w^m + \lambda^m(1-\mu^y)V^o + \lambda^m\mu^m U^o] + \tilde{\rho}_v^m[b^m + \lambda^m U^o]}{\tilde{\rho}_v^m\tilde{\rho}_u^m - \delta^m p^m} \tag{21}$$

$$V^o = \frac{\tilde{\rho}_u^o(w^o + \lambda^o R^\star + \epsilon^o T^\star) + \delta^o(b^o + \lambda^o R^\star + \eta^o T^\star)}{\tilde{\rho}_v^o\tilde{\rho}_u^o - \delta^o p^o} \tag{22}$$

$$U^o = \frac{\tilde{\rho}_v^o(b^o + \lambda^o R^\star + \eta^o T^\star) + p^o(b^o + \lambda^o R^\star + \epsilon^o T^\star)}{\tilde{\rho}_v^o\tilde{\rho}_u^o - \delta^o p^o} \tag{23}$$

$$R^\star = \frac{e^o}{\rho^o + \mu^o} \tag{24}$$

$$T^\star = \frac{d^o + \lambda^{er} R^\star}{\rho^o + \lambda^{er}} \tag{25}$$

for notational convenience we define as follows the adjusted discount rates $\tilde{\rho}^o_u = \rho^o + \delta^o + p^o + \eta^o$, $\tilde{\rho}^o_v = \rho^o + \delta^o + \lambda^o + \epsilon^o$, $\tilde{\rho}^m_u = \rho^m + \delta^m + p^m$, $\tilde{\rho}^m_v = \rho^m + \delta^m + \lambda^m$, $\tilde{\rho}^y_u = \rho^y + \delta^y + p^y$, $\tilde{\rho}^y_v = \rho^y + \delta^y + \lambda^y$.

Profit maximization and *free entry* require that all rents from new vacancy creation are exhausted (i.e. no pure profit in vacancy creation). In other words we have $E^g = 0$, $\forall g$. Substituting these *job creation condition* in equations (12)–(14) gives the following values of a new job for the firm :

$$J^y = \frac{\theta^y}{p^y} c^y \tag{26}$$

$$J^m = \frac{\theta^m}{p^m} c^m \tag{27}$$

$$J^o = \frac{\theta^o}{p^o} c^o \tag{28}$$

these expressions indicate that cost of holding a vacancy equals its expected return.

Stationary equilibrium for expected profit conditions (9)–(11) can also be written as follow :

$$J^y = \frac{a^y - w^y + \lambda^y (1 - \mu^y) J^m}{\tilde{\rho}^y_e} \tag{29}$$

$$J^m = \frac{a^m - w^m + \lambda^m (1 - \mu^m) J^o}{\tilde{\rho}^m_e} \tag{30}$$

$$J^o = \frac{a^o - w^o}{\tilde{\rho}^o_e} \tag{31}$$

where $\tilde{\rho}^y_e = \rho + \delta^y + \lambda^y$, $\tilde{\rho}^m_e = \rho + \delta^m + \lambda^m$ and $\tilde{\rho}^o_e = \rho + \delta^o + \lambda^o + \epsilon^o$.

Let now solve the equations. Labour market are linked : the labour market for "middle-aged workers" depends on "old worker " labour market conditions whereas the labour market for "young workers" depends on "middle-aged workers" labour market conditions.

So we begin by solving the "older aged" labour market. Let first define the inverse labour demand from the firm, from (31) and (28) we have :

$$w^{os} = a^o - \frac{\theta^o}{p^o} c^o \tilde{\rho}^o_e \tag{32}$$

Substitution from (23) and (22) into the solution of the bargaining processes (16) give, under the simplifying assumption that $\epsilon^o = \eta^o$ and knowing that $E^o = 0$ and (31) :

$$w^{od} = \frac{a^o \beta^o (\tilde{\rho}_v^o + p^o) + b^o (1 - \beta^o) \tilde{\rho}_e^o}{(1 - \beta^o) \tilde{\rho}_e^o + \beta^o \tilde{\rho}_v^o + \beta^o p^o} \ , \tag{33}$$

which is the inverse labour supply from the worker. (32) and (33) are function of the market tightness degree θ^o (see (17)) so we can give the following equilibrium value :

$$\theta^o \frac{(1 - \beta^o)\tilde{\rho}_e^o + \beta^o \tilde{\rho}_v^o + \beta p(\theta^o)}{p(\theta^o)} = \frac{(1 - \beta^o)(a^o - b^o)}{c^o} \tag{34}$$
$$\Longrightarrow \sqrt{\theta^o} = \frac{1}{2\beta^o F^o} \left(-[(1 - \beta^o)\tilde{\rho}_e^o + \beta^o \tilde{\rho}_v^o] + \sqrt{[(1 - \beta^o)\tilde{\rho}_e^o + v\tilde{\rho}_v^o]^2 + \frac{4\beta^o F^2 (1 - \beta^o)(a^o - b^o)}{c^o}} \right)$$

Doing the same sequence of calculus for "middle-aged workers" and "young workers" labour markets give us :

$$\sqrt{\theta^y} = \frac{\left(-[(1 - \beta^y)\tilde{\rho}_e^y + \beta^y \tilde{\rho}_v^y] + \sqrt{[(1 - \beta^y)\tilde{\rho}_e^y + v\tilde{\rho}_v^y]^2 + \frac{4\beta^y F^2 (1 - \beta^y)\left(a^y - b^y + \frac{(1 - \_mu^y)\lambda^y J^m}{1 - \beta^m}\right)}{c^y}} \right)}{2\beta^y F^y} \tag{35}$$

$$\sqrt{\theta^m} = \frac{\left(-[(1 - \beta^m)\tilde{\rho}_e^m + \beta^m \tilde{\rho}_v^y] + \sqrt{[(1 - \beta^m)\tilde{\rho}_e^m + v\tilde{\rho}_v^m]^2 + \frac{4\beta^m F^2 (1 - \beta^m)\left(a^m - b^m + \frac{(1 - \_mu^m)\lambda^m J^o}{1 - \beta^o}\right)}{c^m}} \right)}{2\beta^m F^m} \tag{36}$$

We can notice that the value J^o appears in (36) so frictions in the "medium-aged worker" labour market are function of what append in the "old worker " labour market.

2.5 Labour market and demographic flows

2.5.1 Law of motion for labour market flows

Let N be population of the economy, by simplicity and because we only deal with labour market problems we assume that it is the sum of actual working population N^w and of retired N^r and early-retired N^{er} workers eligible for pension benefits of a PAYG retirement scheme ($N \equiv N^w + N^r + N^{er}$). At each date the workers are the sum of worker in the three age groups : $N^w = N^y + N^m + N^o$. For an age group, working population equals to the sum of unemployed and employed people : so $N^g = u^g + L^g$, for $g = y, m, o$.

We assume that there $nNdt$ new "young worker" enter in the labour market during a period dt of time. We assume that there is no movements between working population and inactive population. In the same spirit, $\lambda^r N^r dt$ retired worker are "dying" per period. By definition, the variation of labour and the evolution of unemployed worker are given by :

$$\begin{aligned}
\dot{L}^j &= p^j(\theta^j)u^j - (\delta^j + \lambda^j)L^j && (37)\\
\dot{u}^j &= \delta^j L^j - (p^j(\theta^j) + \lambda^j)u^j + nN && (38)\\
\dot{L}^m &= p^m(\theta^m)u^m + (1-\mu^j)\lambda^j L^j - (\delta^m + \lambda^m)L^m && (39)\\
\dot{u}^j &= \delta^m L^m + \lambda^j(u^j + \mu^j L^j) - (p^m(\theta^m) + \lambda^m)u^m && (40)\\
\dot{L}^o &= p^o(\theta^o)u^o + (1-\mu^m)\lambda^m L^m - (\delta^o + \lambda^o + \epsilon^o)L^o && (41)\\
\dot{u}^o &= \delta^o L^o + \lambda^m(u^m + \mu^m L^m) - (p^o(\theta^o) + \eta^o + \lambda^o)u^o && (42)
\end{aligned}$$

whereas the flows of retired and early retired people are :

$$\begin{aligned}
\dot{N}^r &= \lambda^{er} N^{er} + \lambda^o N^o - \lambda^r N^r && (43)\\
\dot{N}^{er} &= \epsilon^o N^o - \lambda^{er} N^{er} && (44)
\end{aligned}$$

2.5.2 Stationary equilibrium values

At a stationary equilibrium there is no further movements in labour flows. We can obtain from (37)–(44) the following steady-state equilibrium values :

$$\begin{aligned}
\bar{u}^j &= \frac{\delta^j + \lambda^j}{\lambda^j[p^j(\theta^j) + \delta^j + \lambda^j]} nN && (45)\\
\bar{L}^j &= \frac{p^j(\theta^j)}{\lambda^j[p^j(\theta^j) + \delta^j + \lambda^j]} nN && (46)\\
\bar{u}^m &= \frac{(\delta^j + \lambda^j)(\delta^m + \lambda^m) + (\delta^m + \mu^j\lambda^m)p^j(\theta^j)}{[p^j(\theta^j) + \delta^j + \lambda^j]} \frac{nN}{\lambda^m[p^m(\theta^m) + \delta^m + \lambda^m]} && (47)\\
\bar{L}^m &= \frac{(1-\mu^j)p^j(\theta^j)\lambda^m + p^j(\theta^j)p^m(\theta^m) + (\delta^j + \lambda^j)p^m(\theta^m)}{[p^j(\theta^j) + \delta^j + \lambda^j]} \frac{nN}{\lambda^m[p^m(\theta^m) + \delta^m + \lambda^m]} && (48)\\
\bar{u}^o &= \frac{p^o(\theta^o)}{(\lambda^o + \epsilon^o)[p^o(\theta^o) + \delta^o + \lambda^o + \epsilon^o]} nN + \frac{(1-\mu^m)\lambda^m}{[p^o(\theta^o) + \delta^o + \lambda^o + \epsilon^o]} \bar{L}^m && (49)\\
\bar{L}^o &= \frac{\delta^o + \lambda^o + \epsilon^o}{(\lambda^o + \epsilon^o)[p^o(\theta^o) + \delta^o + \lambda^o + \epsilon^o]} nN - \frac{(1-\mu^m)\lambda^m}{[p^o(\theta^o) + \delta^o + \lambda^o + \epsilon^o]} \bar{L}^m && (50)\\
\bar{N}^r &= \frac{\epsilon^o + \lambda^o}{\lambda^r} \frac{nN}{\lambda^o + \epsilon^o} && (51)
\end{aligned}$$

$$\bar{N}^{er} = \frac{\epsilon^o}{\lambda^{er}} \frac{nN}{\lambda^o + \epsilon^o} \quad (52)$$

By construction we have obviously $\bar{N}^j = nN/\lambda^j$, $\bar{N}^m = nN/\lambda^m$, $\bar{N}^o = nN/\lambda^o + \epsilon^o$.

3 Prospects and conclusion

The determination of the stationary equilibrium and the peculiar nature of matching models should enable us to study in depth the transitional impacts of demographic evolution on the labour markets.

The forecast ageing of the population in industrialised countries will have two very separate and distinct components: a decrease in the number of young people and of the fertility rate (ageing by the bottom of the age pyramid), and an increase in life expectancy combined with a decrease of instant death probabilities (ageing by the top). Of course those evolutions don't all affect the composition of the working age population; but, by modifying the ratio of active to inactive people or the conditions for budgetary equilibrium (e.g. pensions), they can impact the labour markets. These evolutions can be described either by an implementation of the evolution of the flux of new entrants or by a modification of the various demographic transition probabilities (the λ^g's and the mortality rate λ^r). Analytically, it is possible to look at an evolution of these last parameters. If an increase in λ^o has the forecast effects (increase in θ^o , in p^o and in w^o), a decrease in λ^y has much more ambiguous effects. The derivatives of the various variables towards this parameter do depend on the values of such variables as the relatives bargaining powers, the job loss probabilities, etc. What is more, an evolution in only these transition parameters do not really describe ageing as it is going to happen. As a matter of fact, a permanent increase in λ^o does more correspond to an increase in work duration than to ageing.

The next step of our work will so consist in a computing implementation of the model. Starting from an equilibrium, we intend to look at the impacts of demographic evolution: the flux of new entrants will be fully described and the λ will be calculated at each period (given the age distribution of the population[1]). This will enable us to look at the evolutions of wages and employment for the three categories of workers. The open nature of this model and the great number of exogenous parameters will enable us to study various scenarii: some budgetary equilibrium conditions can be

[1]For example, if the young category does include the 18-25 years old, λ^j would be equal to 1/7 in equilibrium. The real value of this parameter in transition would correspond to the share of the 24 years old in this category.

added (with taxes) to study the effects of government policies (helps for the hiring of young or old workers; increase in the mandatory retirement age), of ageing on pensions but also on pre-retirement schemes. The model will first be a "static transitional model": that's to mean without any inter-temporal planning done by agents. We intend to add rational anticipations for workers in a second step of this work (implementation under DYNARE shareware). Other directions of research do include at the moment a much more detailed bargaining process (so as to study the real evolutions of seniority premiums), as well as a taking into account of the inactive working age population.

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